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A Statistical Evaluation of AFDC Quality Control

Prepared for:

Office of Family Assistance
Family Support Administration
Washington, D.C. 20201

October 1987

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October 20, 1987

Patrick Brannen
Office of Family Assistance
Family Support Administration
B110 Transpoint Building
2100 Second Street, S.W.
Washington, D.C. 20201

Dear Pat:

This report is submitted in response to contracts 600-84-0262 and HHS-100-87-0009. It is a revision of an earlier version that was submitted for your review and comment on October 2, 1986. However, funds were not available for completing and revising the report until several months later.

This report examines and evaluates the validity of statistical methods in use in AFDC in estimating overpayment errors, and in computing disallowances from those estimates. It does not consider, except for one special case, and otherwise in a very limited way, approximately optimum sample sizes. Also, it does not consider the possible procedures for improving the efficiency of sampling through stratification, through improved allocation of the sample sizes to the state and Federal samples, or other means for reducing sampling errors without increasing total costs -- these will be the subject of a later study.

We note that summaries and details of a number of analyses are included in this report. For most of these, as well as other analyses that have been completed but that are not included in the report, we have diskettes that can be made available for anyone who wishes to obtain the information in this form for further analysis or other purposes.

It has been a real pleasure working with you, and with other OFA staff members in this study. We wish to thank especially, in addition to yourself, Sue Osman, Sean Hurley, John Bowes and Debbie Chassman (who participated before she left OFA), for their many hours of insightful discussion of the issues involved, and for reviewing and commenting on and guiding us in various phases of the study as it has progressed. We also want to express our thanks to Jacqueline Swope Nemes for the outstanding job she did in preparation of the manuscript, and in production of the report.

Sincerely,

Morris

Morris H. Hansen
Chairman of the Board

MH/jsn

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A STATISTICAL EVALUATION OF AFDC QUALITY CONTROL

Prepared by:

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Washington, D.C. 20201

October 1987

A STATISTICAL EVALUATION OF AFDC QUALITY CONTROL

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CHAPTER 1. INTRODUCTION AND SUMMARY

1.1 Introduction

A formal quality control program was introduced in the early 1960's by the Social Security Administration (SSA) to provide guidance in assessing sources of error in the administration of the Aid to Families with Dependent Children (AFDC) program in the various states. The Quality Control (QC) program required each state to institute a review of a sample of cases receiving benefits from AFDC, to carefully reinvestigate these cases and to evaluate the eligibility and amount of the payment made for each sample case, and to provide other information. The principal purpose of the QC review was to identify sources of error, to measure the magnitude of errors to the extent feasible, and to provide information that could guide in taking corrective action. The corrective action could be in the form of improving the administration of the system or of modifying legislation or regulations that were sources of problems.

The state QC sample has been drawn and administered by each state within the framework of the Federal regulations that prescribe and guide the QC program. The program is complicated by the fact that each state has different eligibility requirements and allowances, and the QC administration in a state needs to reflect these differences. Sample sizes in the larger states have been about 1200 cases to be reviewed in each successive six-month period, with smaller samples in the states with small caseloads.[1]

A Federal subsample was drawn from the QC sample in each state to guide and facilitate the administration of QC. The eligibility and the AFDC allowance for the subsampled cases were again intensively reviewed and evaluated. This review provided a framework for improving the quality and comparability of

[1]Optional smaller state sample sizes were recently authorized when QC was placed on an annual basis provided the state signed a statement waiving its right to challenge the validity of the error rate based on the reduced sample size.

administration of the quality control programs although still taking account of the differences in state systems.

Steps were taken in 1973 toward instituting a program of disallowances for states that did not meet a prescribed tolerance, by withholding the Federal share of AFDC payments that were made in error above the allowed tolerance level. This tolerance, which had been administratively established by the Department of Health and Human Services, was subsequently set aside by the Federal District Court as lacking an empirical basis. In 1980, the Congress established decreasing tolerances to be attained in fiscal years 1981 and 1982, with a tolerance of 4 percent in the overpayment error rate in 1983. The 4 percent tolerance was reiterated in the Tax Equity and Fiscal Responsibility Act (TEFRA) of 1982, which established a 3 percent tolerance for fiscal year 1984 and thereafter. Consequently, an important goal of the Federal QC sample review, in addition to providing guidance for improving program administration and design, became to provide estimates of overpayment error rates for the purpose of determining the amount of any disallowances.

It would be possible to estimate the overpayment error rates directly from the results obtained from the Federal subsample, without making use of the available state QC results. However, Westat recommended a double sampling procedure for drawing the Federal subsamples from the state samples and for preparing estimates. This procedure produced considerably more precise results from a given size of Federal subsample, and was adopted.

More specifically, a regression estimator was recommended by Westat, in memoranda dated June 18, 1973 and July 19, 1973.[2] The regression estimator with double sampling makes use of the results available from the full quality control sample for a state, together with those for the Federal subsample. Its use, in practice, has generally had the effect of reducing the sampling variance of estimates of payment error rates by about 50 percent or more, as compared with using only the Federal subsample. Stated in a different way, the double sampling plan and estimator generally yield results equivalent in precision to what would have been

[2]Memorandum submitted by Morris Hansen, Westat, to John C. Young, Social and Rehabilitation Service, DHEW, ***Review and evaluation of proposed use of QC system for Federal estimates of ineligible and overpayment cases***, June 18, 1973, and supplemental memorandum dated July 19, 1973.

obtained by at least doubling the size of the Federal subsample and basing the estimate only on the Federal findings. However, if the Federal sample cases were not reviewed by the state in advance of the Federal review, the quality of the Federal review would be adversely affected, and its cost considerably increased, since in the present procedure the Federal reviewer has an easier job (and presumably does a better job) because s/he has the advantage of the previous state review. Thus, to maintain the same quality without the use of the double sampling estimator, not only would the Federal sample have to be increased by a factor of two to three, but the sample would also need to be reviewed by the state. For example, if the state sample size is now 1200 and the Federal sample 360 (giving a total of 1560 reviews), doubling the sample would mean state and Federal samples of 720 (giving a total of 1440 reviews). This would reduce the cost of the QC reviews by only 8 percent (assuming about equal costs for the state and the Federal review). If the Federal sample size had to be somewhat more than doubled to get the same precision, as is likely, the cost would actually be increased. Even more important is the fact that reducing the size of the state sample in this manner would greatly reduce the effectiveness of the QC program in its primary goal, that of identifying causes of error and guiding appropriate corrective actions.

It should be noted that the double sampling and regression estimation procedure does not "adjust" the state estimates -- instead, it provides estimates of what would result if the Federal QC review, preceded by a state review, were applied to the entire caseload. It is simply a procedure for reducing the sampling error of the estimate from the Federal subsample. It makes use of the fact that the Federal and state findings on individual cases are highly correlated. Consequently, if the overpayment errors based on state findings for the cases in the Federal subsample are above those in the full state sample, then the Federal findings based on that sample are likely also to be too high. The regression estimator adjusts for the difference in average state findings in the two samples. A similar sampling error adjustment results if the state findings in the Federal sample are below the state findings in the full state sample. Thus, by use of the regression estimator, the effective sample size of the Federal subsample is increased substantially since there is a high correlation of case-by-case findings from the state and the Federal reviews. The estimate based on the Federal review in a state may or may not agree with the state estimate, depending on the amount of agreement between individual Federal

and state case findings. Thus, the results from the regression estimator are estimates of what would be obtained if the state QC review, followed by the Federal review, were applied each month to all cases receiving AFDC. Of course, such a procedure would be prohibitively costly.

As currently used in AFDC, the regression estimator of the overpayment error rate (referred to also as the payment error rate) for any given state is

$$\hat{R} = \frac{\bar{x}''}{\bar{t}} = \frac{\bar{x}' + b(\bar{y} - \bar{y}')}{\bar{t}} \qquad (1)$$

where

$\bar{x}' = \sum^{n'} x_i/n'$ is the average overpayment error per case in the Federal subsample as determined by the Federal review (it is the average over all cases whether or not there was an overpayment error involved);

$\bar{y} = \sum^{n} y_i/n$ is the average overpayment error in the state QC sample as determined by the state review;

$\bar{y}' = \sum^{n'} y_i/n'$ is the average overpayment error as determined by the state QC review for the cases included in the Federal subsample;

$\bar{t} = \sum^{n} t_i/n$ is the average AFDC payment for the cases in the state QC sample;

$$b = \frac{\sum^{n'} x_i y_i - n'\bar{x}'\bar{y}'}{\sum^{n'} (y_i - \bar{y}')^2} \qquad (2)$$

is the regression coefficient estimated from the Federal subsample;

n is the size of the state QC sample;

n' is the size of the Federal subsample;

x_i, y_i, and t_i are, respectively, for the i-th case in the designated state or Federal sample, the amount of overpayment as determined in the Federal review, the amount of overpayment as determined in the state QC review, and the AFDC payment for the case;

$$s_{\hat{R}} = s_X \left\{ \left[1 - r^2 \left(\frac{n - n'}{n} \right) \right] / n' \right\}^{1/2} / \bar{t} \tag{3}$$

is the estimated standard error of $\hat{R}$;

$r = b \frac{s_Y}{s_X}$ is the coefficient of correlation of x_i and y_i, estimated from the Federal subsample;

$$s_X = \left\{ \sum^{n'} \left(x_i - \bar{x}' \right)^2 / \left(n' - 1 \right) \right\}^{1/2}$$

is the unit standard deviation of the payment errors as determined in the Federal review and as estimated from the Federal subsample;

$$s_Y = \left\{ \sum^{n'} \left(y_i - \bar{y}' \right)^2 / \left(n' - 1 \right) \right\}^{1/2}$$

is the unit standard deviation estimated from the Federal subsample of payment errors as determined in the state review.

The above and other formulas used (except as otherwise specified) assume simple random sampling of the state QC sample from the file of AFDC payment records, and of the Federal subsample from the state QC sample. In practice, in most states the samples are drawn by proportionate stratified systematic sampling procedures rather than simple random sampling. The stratification is by months, with the same fraction of cases sampled each month. The systematic selection within months ordinarily involves taking every k-th case from an ordered list with a random start and with the ordering likely to involve geographic or alphabetic sequencing, or both. Simple random sampling formulas are commonly

applied in such situations, and in this application they should give quite good approximations.[3] In a few states, other modes of stratification are sometimes used.

In the original memoranda recommending the use of the regression estimator to estimate the overpayment error rate and its standard error, $\bar{T}$ was used in the denominator instead of $\bar{t}$, where $\bar{T}$ is the average payment per case for the total AFDC caseload for the period. It turned out that $\bar{T}$ was not reasonably available in practice, and $\bar{t}$ has been substituted. As indicated later in Appendix I, this substitution has been quite satisfactory.

A question that has concerned us about these estimators is that the regression estimator and its estimated standard error are based on approximations that hold for large enough samples, but that may not be reasonably acceptable for samples of the sizes used for the Federal subsample in some or all of the states. The size of the Federal subsample for a six-month period has varied generally between about 70 and 200 cases for the various states, and thus between about 140 and 400 cases for a full year. Ordinarily, samples of these sizes would not be considered too small if the samples were drawn from populations that are not extremely skewed. However, the populations in this case are extremely skewed, with no payment errors found in about 80 to 90 percent of the cases, and with considerably varying and highly skewed payment errors occurring in the remaining 10 to 20 percent of the cases.

Because of this concern, in a later memorandum[4] concerning the QC program in Supplemental Security Income (SSI), we recommended, on the basis of a preliminary evaluation, the substitution of a difference estimator for the regression estimator. The difference estimator is of the same form as the regression estimator except that a constant, k, is substituted for b (b is estimated from the sample and is

[3]We have compared such stratified sampling with simple random sampling for the Food Stamp QC program, which is similar to the AFDC-QC program, and found remarkably close agreement of results for the two procedures (i.e., simple random sampling and stratified proportionate sampling by months).

[4]Memorandum dated September 30, 1981, submitted by Westat to Social Security Administration, Office of Payment Eligibility and Quality.

subject to sampling variability). The regression estimator is evaluated in Section 2.2, where it is shown to provide unbiased or at most trivially biased estimates. The difference estimator is evaluated for AFDC-QC in Appendix B, and compared with the regression estimator. This evaluation shows little difference between the two estimators and leads us to conclude that we see no advantages to AFDC in changing to the difference estimator.

Some of the states have argued that if disallowances are to be imposed they should not be computed on the basis of the point estimate, as now prescribed. They suggest that since the overpayment error rates are based on samples, a lower confidence bound should be used, e.g., a bound computed for the sample such that there is a low probability that the lower bound of the confidence interval computed for each of the possible samples is less than the true error rate, and a high probability that it is greater.

Such an approach would, on the average, systematically and substantially underestimate the amount which would be disallowed if the true error rate were known. The state's gains would be the Federal government's loss. Moreover, the amount of the disallowances would depend importantly on the sample size (the disallowance for a state would be less for a given error rate, on the average, if a smaller QC sample size were used). Also, a problem arises because a state could lower the confidence bound by inadvertently or deliberately doing lower-quality work in the state QC, thus increasing the sampling error of the regression estimate of the payment error rate. This is because a reduction in the quality of the state QC results would increase the number of discrepancies between the state and Federal evaluations. These increased discrepancies would decrease the correlation between the state and the Federal findings, and thus (as can be seen from Equation (3) above) would increase $s_{\hat{R}}$, the estimated standard error of the regression estimator. Since, for example, a 95 percent nominal lower confidence bound is computed by subtracting $1.645 s_{\hat{R}}$ from the estimated error rate, the result would be a lower average value for the computed lower confidence bound and, hence, a smaller disallowance. Consequently, there might be an incentive for a state to lower the quality of work, in order to avoid or reduce disallowances.

We note (as discussed in Section 3.3 and in Appendix D) that a minor change in the standard procedure for computing lower confidence bounds would substantially eliminate this problem. This procedure involves assigning a minimum value for the correlation of Federal and state findings (a minimum rho) in estimating the variance.

While more research is desirable, we have made enough progress that some guidance is provided in this report on the first two of the following important questions that you have asked us to examine. These questions include the following:

- Are the sampling procedures and the regression methodology used by the AFDC-QC statistically valid?
- What are the considerations and constraints involved in the choice of a lower confidence bound versus a point estimate in determining disallowances?
- What are the considerations and constraints in the choice of sample size for the state quality control samples and for the Federal review samples?
- Are there any means of decreasing the sampling errors (and reducing the width of confidence intervals) of estimated state error rates other than by increasing sample size?

In the following sections of this report, we provide some answers to the first two of these questions in as nontechnical language as feasible, on the basis of the work that has been completed. Fuller technical analyses and more detailed considerations of some of the issues and the implications of alternatives are included in the relevant appendices. Some very limited preliminary attention is given in this report to the last two questions. They will be more fully considered in a second report.

Before proceeding to the more detailed discussion, we provide a summary of the principal conclusions from the work that has been done.

1.2 Some Summary Results and Conclusions

On the basis of the evaluation work that has been completed, we are able to summarize the results and conclusions as follows:

(1) The procedures specified for drawing the state samples and the Federal subsamples are applications of standard and widely used sampling methods, and if the samples are made large enough, they will yield estimates of overpayment error rates as close as desired to the value being estimated. The value being estimated is defined as the expected value that would be obtained if the entire caseload were reviewed by both state and Federal reviewers (as is done for the Federal subsample).

(2) The regression methodology for making estimates from the samples provides statistically valid estimates, unbiased in the sense that, on the average over all possible samples that could be drawn by the specified procedures for a state, the regression estimate of the overpayment error rate is equal or very nearly equal to the value being estimated. This statement holds for each of the differing sample sizes in use in the various states. Moreover, as sample size increases, the sampling errors of the regression estimates decrease, and consequently the estimates are closer, on the average, to the value being estimated.

(3) The sample estimates of the variance of the estimates of overpayment error rates are also, on the average, reasonably close to the variance over all possible samples, and the computed sampling errors or confidence intervals provide, on the average, acceptable measures of precision. However, the sampling errors of the direct state variance estimates are so large that the use of the estimated variance from a single state sample for purposes of estimating needed sample sizes to achieve specified levels of precision, or to provide general measures of precision, can yield exceedingly variable and misleading results. In Section 2.5 a pooled variance estimation procedure is developed and presented that greatly improves the variance estimates for such uses.

(4) Classical regression analysis requires the assumption of a linear relationship between the dependent and the independent variables, and normal

distributions of the dependent variable for given values of the independent variable(s). The use of the regression estimator in estimating AFDC overpayment errors has been widely challenged on the grounds that the assumptions of classical regression are grossly violated. However, these challenges do not recognize the difference between classical regression analysis and the application of the regression estimator in sample surveys, as in AFDC-QC. For such applications, the assumptions are not required. Mathematical proof of the validity of the application of the regression estimator in sample surveys with sufficiently large samples, independent of the distribution from which the samples are drawn, is given by Cochran in a classical paper on regression estimation in sample surveys.[5] In addition to that proof, we provide a number of examples involving different AFDC-QC populations and sample sizes illustrating the fact that the application of the regression estimator in AFDC for sample sizes similar to the sample sizes in use does yield valid results, as described in points (1) through (3) above (see Section 2.2 and Appendix B). These illustrative results are provided for each of four sample sizes for each of three illustrative test populations based on actual AFDC data.

(5) We also note that in the application of the regression estimator to AFDC, the regressions involved are of sample means rather than of the original observations and the relationships between the sample means are indeed closely linear. Also, while the conditional distributions of the dependent variable for any given value of the independent variable are slightly skewed, they are reasonably close to normal (see Section 2.2). Consequently, although meeting the classical assumptions is not necessary, they are in fact reasonably met in the application of the regression estimator in AFDC Quality Control.

(6) The distributions of individual case overpayment errors are highly skewed. Consequently, the nominal 95 percent confidence intervals which are now computed from the samples on the assumption of normal distributions are imperfect. If the distributions of overpayment errors were normal, then, on the average in repeated samples, for the sample sizes in use, close to 2-1/2 percent of the time the value being estimated would be below the computed 95 percent confidence

[5]Cochran, W.G., ***Sampling Theory When the Sampling Units are of Unequal Size***, Journal of the American Statistical Association, Vol. 37, pp. 199-212, 1942.

interval and close to 2-1/2 percent of the time it would be above. In fact, the "tails" above and below the confidence intervals of the overpayment error rate estimates depart considerably from these expectations. For considerably less than 2-1/2 percent of the samples the lower confidence bound is above the value being estimated, and for considerably more than 2-1/2 percent of the samples the upper confidence bound is below the value being estimated. The combined effect is that confidence intervals cover the values being estimated with somewhat less than the nominal 95 percent probability. Thus, the precision actually achieved is somewhat less than would be the case if the 95 percent confidence were actually achieved. Nevertheless, the 95 percent (or 90 percent) confidence intervals provide reasonably satisfactory indicators of precision. It is important to note that the estimates of overpayment error rates are unaffected by any imperfections in the computed confidence intervals.

(7) We have developed and have done some testing of an improved method for computing confidence intervals that will yield considerably closer approximations to the nominal probabilities. The results appear in Section 2.4 and in Appendix C.

(8) The decision on whether to use point estimates or lower confidence bounds in determining disallowances is a policy one, and depends on the goals to be served. There are precedents for both approaches, as discussed in (12) through (13) below.

(9) If the goal is to approximate the true disallowance, i.e., the disallowance that would be made if the true overpayment error rate were known, the point estimate satisfies the goal. Business organizations use sampling with point estimates to settle the sharing of large costs or benefits, as in the distribution of funds from jointly furnished services (for example, the distribution of funds by the railroads from shipments that go over two or more lines), or as in the sharing of joint costs (for example, joint maintenance costs of poles used to carry both telephone and electric cables). Similarly, sample surveys with point estimates are widely used in establishing rate bases for utilities (for example, to estimate replacement cost of plant and equipment from inspections of samples of such equipment) and in many other applications. Such applications of samples and the

point estimate generally call for samples large enough to yield reasonably precise estimates.

(10) Computation of annual disallowances from QC samples are commonly subject to relatively large sampling errors, especially if payment error rates are less than about 4 percentage points above tolerance. Sampling errors of disallowances can be as much as 50 to 100 percent or more of a single year's disallowance. This problem could be substantially eliminated by making some modifications in the way disallowances are administered, so as to take fuller advantage of compensations over time (see Section 3.7).

(11) If the goal is to assess disallowances separately for each year and then only to the extent that they have been reasonably proved to be at least a specified amount or more, then a lower confidence bound satisfies the goal. It is common in auditing, for example, to follow up leads of evidence of possible fraud from sample audits only if a lower confidence bound of an estimate is exceeded.[6]

(12) Use of the lower confidence bound would, on the average, result in AFDC disallowances that are much less than they would be if the true overpayment error rates were known and used in computing disallowances. The Federal government would absorb the loss, and this loss would be substantial. Consequently, if lower confidence bounds were to be adopted for computing disallowances, cost-benefit considerations indicate that, for states in which large disallowances are involved, it would be advantageous to the Federal government to use considerably larger samples than those now used (see Sections 3.4 and 3.5). Increases in state samples may also be called for.

(13) The determination of appropriate sample sizes for QC for purposes of evaluating and guiding improvements in the AFDC program involves difficult issues, and there are no simple answers. Some limited preliminary discussion of these issues appears in Chapter 3.

[6]See, for example, Arkin, Herbert, ***Sampling Methods for Auditors***, McGraw-Hill Book Company, New York, pp. 56-58, 107-109.

(14) We see no obvious striking gains to be achieved by modifications in the design of the QC samples other than by increasing the state or Federal sample sizes. However, some gains may be feasible. Our explorations to date in this area are quite limited, and further work is needed in order to evaluate any such potential gains.

(15) We add a final remark on a topic that we believe should be mentioned here. It has sometimes been suggested that the primary role of the QC samples should be to determine disallowances, and that corrective action inferences could better be guided by other special analyses and studies. Such a separation seems to be unnecessarily costly and undesirable. We anticipate that it may be possible to increase the effectiveness of the QC sample by subjecting the data to discriminant analyses, cluster analyses, or other methods of error-prone profiling, and thereby identify subclasses that contribute a high proportion of errors. Such studies could lead to the introduction of more effective stratification and more efficient allocation of the samples. The next phase of our study will include examining such methods for improving precision without increasing sample size. Thus, if error-prone profiling proves to be effective, it could also help provide the much-needed improvements in precision of the QC sample when used for assessing disallowances. At the same time, it would also increase its effectiveness for analyses of sources of error and feedback for corrective action, and may also prove to be an effective tool for improving case reviews in administration. To separate the two uses would only add to cost and decrease performance.

We note also that other sources of data such as income tax matching, wage matching, or bank matching have been suggested as an alternative to quality control reviews. Such data can be very useful, to the extent that their use is cost effective, in improving the administration of AFDC. Evaluation and possible extension of such uses are part of the current program of the Office of Family Assistance (OFA). These procedures do not replace the need for QC, but to the extent that they lower error rates, they may reduce the need for corrective action and may also reduce disallowances. After sufficient reduction in error rates has been accomplished in a state, then a reduction in the size of the QC sample would be appropriate in that state -- but the sample must still be large enough to monitor for early detection of a serious deterioration of quality.

CHAPTER 2. STATISTICAL VALIDITY OF AFDC-QC METHODOLOGY

We first address the question:

- *Are the sampling procedures and the regression methodology used by the AFDC-QC statistically valid?*

We have examined the specified sample selection and estimation procedures, and have reviewed existing theory and in some cases extended the theory. The available theory is not exact but holds for large enough samples. However, available statistical theory does not tell us what size samples are large enough; that is, what size samples are needed to achieve sufficiently close approximations. Consequently, we have done extensive simulations by drawing large numbers of independent samples from three test populations and prepared estimates from them for alternative sample sizes for each of the populations. The test populations, described in Appendix A, are samples of actual AFDC-QC cases. Many of our conclusions are based on the results of these simulations.

In the balance of this report, we discuss more fully and illustrate the basis for most of the summary remarks that appear at the end of Chapter 1, and provide some extensions of them.

2.1 Test Populations

To examine the accuracy of the approximations, we have done extensive testing with three test populations (referred to as Populations A, B, and C) using actual AFDC-QC data from the Federal subsamples for the year ending September 30, 1982.

Population A was created by taking the state and Federal QC results for the cases included in the Federal subsample for Illinois, New Jersey, Ohio, and Pennsylvania. These were four large states that had roughly similar average payments for AFDC and roughly similar average overpayment error rates.

Population B used the state and Federal QC results for cases included in the Federal sample for Texas, South Carolina, Maryland, and Michigan. These are relatively large states with somewhat different characteristics from those of Population A.

Population C used the state and Federal QC results for cases included in the Federal subsample for six states with relatively smaller AFDC-QC sample sizes, including Arkansas, Colorado, Hawaii, Nebraska, Oregon, and West Virginia.

Some of the characteristics of the three test populations and of the AFDC results for all states for the six-month period ending September 1982 are summarized in Table 2-1 and more fully in Appendix A.

Various tests were carried through by drawing 1000 independent samples of each of a number of specified sample sizes from these test populations, and computing and evaluating various estimates from these samples. Among the sample sizes used in evaluating the regression methodology were the following:

	Annual sample size			
	1	2	3	4
Size of state sample, n	2400	1200	880	350
Size of Federal subsample, n'	360	360	260	160

Each of the state samples was obtained by drawing with replacement from the population a simple random sample of the specified size, and then drawing a simple random sample without replacement from the state sample for the Federal subsample. Drawing the state sample with replacement has the effect of making the simulation process equivalent to drawing the sample from a much larger population, and in effect, simulates the drawing of the state sample from a very large state AFDC population equivalent in composition to the test population.

Table 2-1. Some characteristics of the test populations and of the full AFDC population (1982)

	Units	Test Population A	Test Population B	Test Population C	Average U.S. 6 months ending September 1982
Average AFDC payment ($\bar{T}$)	dollars	296	210	255	302
Standard deviation of payments	"	255	121	194	n/a
Overpayments					
Average based on Federal review	"	21.6	15.0	16.9	20.
Average based on state QC review	"	17.2	16.7	13.7	n/a
Unit standard deviation of overpayments Federal review	"	70.5	58.6	66.1	n/a
Correlation of Federal and state overpayments	--	0.83	0.94	0.81	0.85*
Overpayment rate (Federal review)	percent	7.30	7.95	6.62	6.64
Percent with overpayments (Federal review)	percent	12.7	13.1	11.2	15.2

n/a - Not readily available.

*Simple mean of the estimates for the 45 states that did not treat their samples as stratified samples for the state QC during this period (the mean was roughly the same for the remaining states).

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Table 2-2 shows state and Federal AFDC-QC sample sizes by state, for the year ending September 30, 1982. Sample sizes 1 and 2 above correspond approximately to and are illustrative of the sample sizes used in about 24 of the larger states. Sample sizes 3 and 4 are illustrative of samples used in a number of medium-sized and smaller states.

2.2 Evaluation of the Regression Estimator

Classical regression analysis is based on the assumption of a linear relationship between the dependent and the independent variables, and on the assumption that the dependent variable is approximately normally distributed for each value of the independent variable. However, as we have noted in Section 1.2, the fact that the joint distribution of individual state and Federal case findings of payment errors fails to satisfy these assumptions is not relevant for the choice of an estimator. As can be seen from Equation (1), (Section 1.1), the regression estimator depends, not on the relationship of state and Federal findings of error for the individual cases, but on the relationship of the sample means of those findings in the Federal subsample. Based on 1000 independent samples from each test population for each of four sample sizes, it is clear that the relationship between the means is closely linear. Figure 2-1 shows scatter diagrams of the relation of the sample mean of Federal findings and the sample mean of state findings for the same sample, for 1000 samples drawn from Test Population A for each of four different sample sizes.[1] It is clear from the diagrams that there is little if any departure from a linear relationship. Also, the distributions of the points about the fitted lines are approximately although not quite normal. Thus, the assumptions of classical regression analysis are fairly well satisfied. We emphasize again, however, that although the classical assumptions appear to be reasonably well satisfied, meeting them is not required in order to assure the validity of the regression estimator. Rather, that validity requires only that the variances and covariance involved are finite, and that the sample is sufficiently large (see Cochran, op. cit., p. 203, and see also Appendix B). Since the first of these conditions is obviously satisfied when sampling from a finite population such as the AFDC case determinations, it remains only to ask if the samples used in AFDC-QC are large enough. It is for this purpose that we examine the results of sampling from test populations made up of real data, using sample sizes that approximate those in actual use.

[1] Similar diagrams for two other test populations are included in Appendix B.

Table 2-2. Sample sizes by state for 12-month period ending September 30, 1982. [Samples are treated as stratified samples in some states, with stratum figures shown in parentheses ( ).]

State	State sample n	Federal sample n'	State	State sample n	Federal sample n'
Alabama	2,211	377	Michigan	2,396	361
Alaska	314	134	Mississippi	1,995	365
01	(225)	(96)	Minnesota	1,718	311
02	(89)	(39)	Missouri	2,580	389
Arizona	748	229	Montana	330	156
Arkansas	1,070	301	Nebraska	424	183
California	2,432	366	Nevada	329	152
Colorado	908	274	New Hampshire	295	137
06	(129)	(40)	New Jersey	2,358	362
07	(655)	(193)	New Mexico	636	208
61	(33)	(8)	New York	2,483	364
62	(91)	(33)	North Carolina	2,422	368
Connecticut	1,733	356	North Dakota	346	160
Delaware	304	167	Ohio	2,491	386
District of Columbia	938	266	Oklahoma	1,409	298
Florida	2,534	394	Oregon	1,174	285
Georgia	2,445	376	Pennsylvania	2,466	375
Hawaii	605	210	Rhode Island	625	211
Idaho	334	129	South Carolina	2,431	376
Illinois	2,381	358	01	(1,221)	(175)
01	(339)	(47)	02	(1,210)	(201)
02	(1,478)	(223)	South Dakota	326	151
03	(564)	(88)	Tennessee	2,157	359
Indiana	2,063	364	Texas	2,399	374
Iowa	1,208	304	Utah	323	172
Kansas	776	242	Vermont	301	156
Kentucky	2,137	364	Virginia	2,330	358
Louisiana	2,421	382	Washington	1,942	341
Maine	631	218	West Virginia	971	273
Maryland	2,425	365	Wisconsin*	2,508	394
Massachusetts	2,401	354	01	(1,704)	(266)
00	(1193)	(175)	02	(804)	(128)
01	(594)	(92)	Wyoming	339	168
02	(614)	(87)			

*Figures quoted are twice those for the last 6 months of the year.

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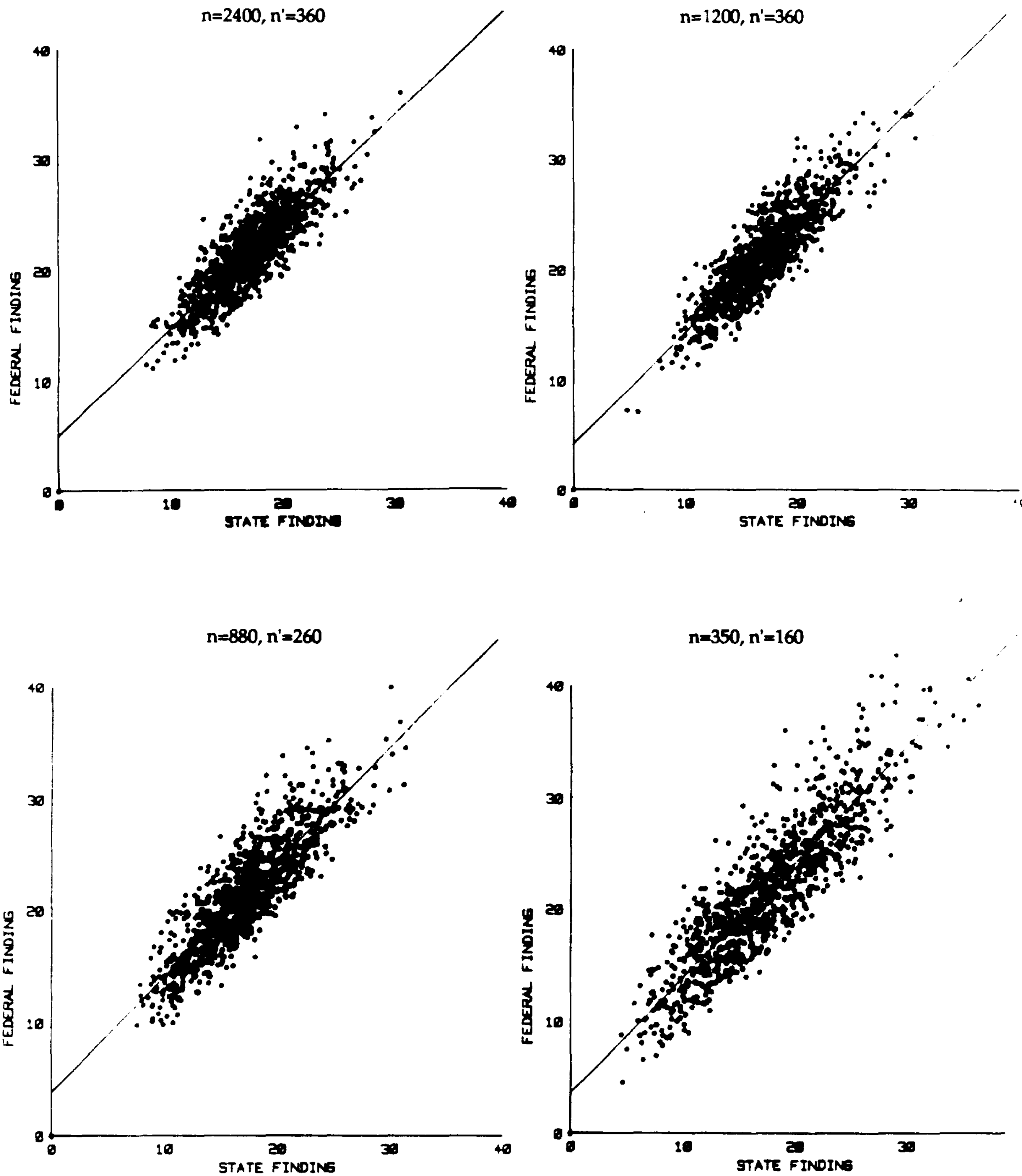
Figure 2-1. Mean findings of dollar error per case in 1000 independent samples for each of four sample sizes, Population A



Some of the results based on replicate samples drawn from Population A are summarized in Table 2-3. Similar results were obtained for the other test populations and are presented in Appendix B. These results indicate that for the various sample sizes in use the regression methodology provides valid estimates of overpayment error rates for the various sizes of annual state and Federal samples in use. By valid estimates, we mean that for a given sample size the average of the estimates over a large number of samples is close to the value being estimated, and that the computed sampling errors or confidence intervals provide approximate but acceptable indicators of precision.

Illustrations are provided by comparing lines 1 and 2 of Table 2-3 and also by comparing the differences between these (line 3) with their estimated standard errors (line 4). For each sample size, the average of the overpayment error rate estimates is closely equal to the overpayment error rate in the test population. Similar results are seen from the additional comparisons available in Table B-3 of Appendix B. While the estimates are almost all less than the population values, the differences are all far less than their sampling errors. All such differences contribute less than one percent to the estimated mean square errors of $\hat{R}$. We conclude that here is a trivial negative bias in the regression estimator. Any such bias decreases faster than the sampling error decreases as sample size is increased.

Table 2-3 also illustrates that, with the regression methodology applied to Test Population A, the estimated variances of $\hat{R}$ (line 6) are all reasonably close to the estimated true variances (line 5). The differences are all small relative to their estimated standard errors. Again, similar results are seen in Table B-3 of Appendix B for Test Populations B and C.

2.3 Evaluation of Computed Confidence Intervals

Another way to examine the validity of the regression methodology is to determine, for example, the proportion of times in repeated sampling that the computed nominal 95 percent or 90 percent (two-tailed) confidence intervals include the true payment error rate, and the proportion of times that the true payment error rates are above or below the specified nominal confidence bounds. Such results are shown in Table 2-4.

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Table 2-3. Evaluation of regression estimator based on computations for 1000 independent samples drawn from Test Population A

Statistic	Sample size (n and n')			
	1 2400 360	2 1200 360	3 880 260	4 350 160
1. True overpayment error rate in test population	.0730	.0730	.0730	.0730
2. Average of estimated overpayment error rates from 1000 samples ($\bar{R} = \Sigma\, \hat{R}_k/1000$)	.0731	.0724	.0727	.0729
3. Difference (Line 1 – Line 2)	-.0001	.0006	.0003	.0001
4. Estimated standard error of difference (standard deviation of $\hat{R}_k$ from 1000 samples)	.00025	.00027	.00033	.00048
5. Estimated true variance of $\hat{R}$ based on variance of $\hat{R}$ from 1000 samples $\hat{\sigma}^2_{\hat{R}} = [\Sigma\, (\hat{R}_k - \bar{R})^2/1000]\ (\times 10^4)$	.628	.704	1.073	2.29
6. Average of estimated variances of $\hat{R}$ from each of 1000 samples $av(s^2_{\hat{R}_k}) = [\sum_k s^2_{\hat{R}_k} / 1000]\ (\times 10^4)$	.645	.799	1.100	2.19
7. Difference (Line 5 – Line 6)	-.017	-.095	-.027	.10
8. Estimated standard error of difference (Line 7)*	.031	.109	.053	.113
9. Standard error of estimated variances of $\hat{R}$ $[\Sigma\, (s^2_{\hat{R}_k} - av(s^2_{\hat{R}_k}))^2 /1000]^{1/2}\ (\times 10^4)$	.22	.23	.39	.87

*Computed from $\sigma^2_{diff} = \frac{1}{1000}\{(\text{Standard error of estimated variance of } \hat{R})^2 + (\hat{\sigma}^2_{\hat{R}})^2 (\beta - 1)\}$

with β assigned the value 3.3. Essentially the same results would have been obtained for β assigned values from 3 to 4, which seem reasonable from Figure C-1 in Appendix C. Direct estimates of β varied between 2.8 and 3.2. The value 3.3 was taken as an approximation before the direct estimates were available, and was so close that it was not worth recomputing.

Table 2-4. Proportion of observed samples in which value being estimated was above, below, or covered by specified nominal confidence bounds, for Test Populations A, B, and C

Nominal confidence bound	Test Population	Sample sizes			
		2400/360	1200/360	880/260	350/160
Below .025 point	A	.011	.006	.010	.013
	B	.011	.012	.008	.017
	C	.003	.011	.009	.007
	Average	.008	.010	.009	.012
Below .05 point	A	.024	.028	.028	.031
	B	.032	.030	.033	.036
	C	.014	.021	.020	.028
	Average	.023	.026	.027	.032
Above .95 point	A	.084	.097	.100	.102
	B	.093	.072	.093	.096
	C	.093	.103	.113	.120
	Average	.090	.091	.102	.106
Above .975 point	A	.053	.059	.066	.075
	B	.067	.042	.055	.062
	C	.060	.080	.084	.087
	Average	.060	.060	.068	.075
Between .05 and .95 points	A	.892	.875	.872	.867
	B	.875	.898	.874	.868
	C	.893	.876	.867	.852
	Average	.887	.883	.871	.862
Between .025 and .975 points	A	.936	.935	.924	.912
	B	.922	.946	.937	.921
	C	.937	.909	.907	.906
	Average	.932	.930	.923	.913

*Based on 1000 independent replicate samples for each sample size for each test population.

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The nominal 95 percent confidence intervals (and other confidence intervals) as now computed for AFDC-QC make use of normal distribution theory, i.e., assume that the distribution of the estimated payment error rate and its estimated standard error are distributed approximately as they would be for an estimated mean based on simple random samples of about 30 or more observations drawn from a normal distribution. Thus, the 95 percent confidence intervals are computed for the overpayment error rate, $\hat{R}$, by computing $\hat{R} \pm 1.96 s_{\hat{R}}$, where $s_{\hat{R}}$ is the estimate from the sample of the standard error of $\hat{R}$. For large enough samples drawn from the AFDC population of overpayment errors, the probability that such a confidence interval will cover the true value will be reasonably close to the nominal 95 percent. We refer to this as the nominal probability. If the overpayment errors were normally distributed, then, on the average, approximately 95 percent of such confidence intervals would include the value being estimated, and in about 2-1/2 percent of the samples the lower bound would be below the value being estimated, and in about 2–1/2 percent of the samples the upper bound would be above.

In AFDC-QC, as illustrated in Table 2-5 for Test Population A, the distribution of overpayment errors is a very skewed rather than a normal distribution. Also, AFDC-QC uses a double sample and a regression estimator. To help evaluate the usefulness of the computed confidence intervals under these circumstances, we have examined how close the observed probabilities are to the nominal probabilities. We have done this by taking repeated independent samples from each of the three test populations described in Section 2.1 and more fully in Appendix A.

From Table 2-4, it is seen that for each test population and, on the average over the three test populations, the fractions for which the true value was *below* the nominal 95 percent two-tailed confidence intervals is considerably less than the 2-1/2 percent that would be expected if the samples were drawn from normal distributions. Conversely, R was above the computed confidence intervals in a considerably *higher* fraction than the nominal 2-1/2 percent. More specifically, on the average for the three test populations, for each sample size the value being estimated falls below the lower nominal 95 percent confidence bound for only about 1 percent of the samples, and in about 6 to 7 percent of the cases it falls above the upper nominal confidence bound. The differences between these percentages and the 2-1/2 percent nominal percentage cannot be explained by sampling variability.

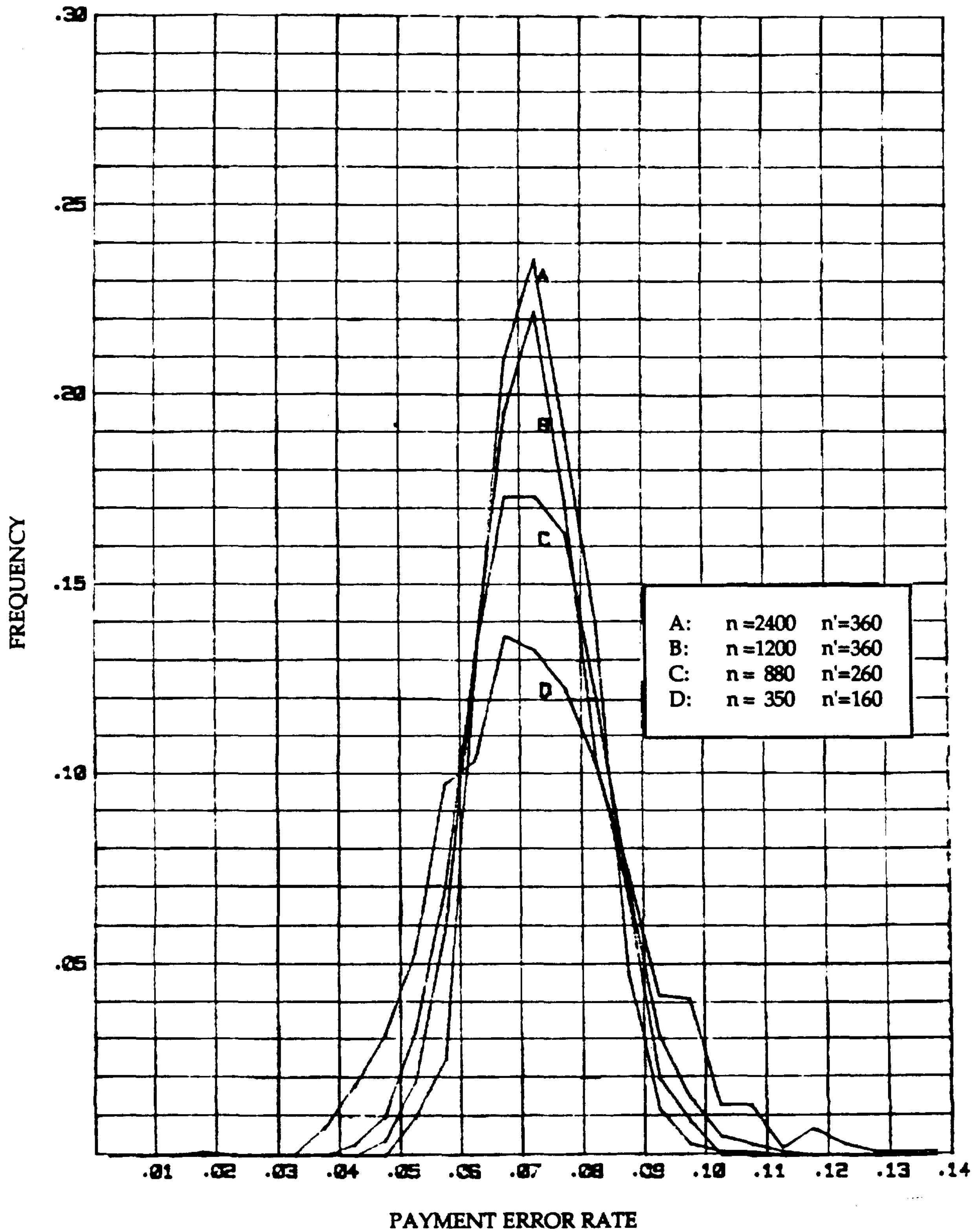
Table 2-5. Percentage distribution of overpayment errors as determined by the state and Federal evaluation for Test Population A (Note that in this table, as in the analyses, underpayment errors are treated as zero overpayment errors.)

Overpayment errors ($) per state QC	Overpayment errors ($) per Federal QC							
	None	1-99	100-199	200-299	300-399	400-499	500-599	Total
None	86.7	0.5	0.6	0.6	0.3	0.2	0.1	89.0
1-99	0.4	4.4	--	0.1	--	--	--	5.0
100-199	0.1	--	2.0	--	--	--	--	2.1
200-299	0.1	--	--	2.2	--	--	--	2.2
300-399	0.1	--	--	--	1.5	--	--	1.6
400-499	--	--	--	--	--	0.1	--	0.1
Total	87.3	4.9	2.6	2.9	1.8	0.3	0.1	

Table 2-4 also shows that for the largest sample size (n=2400, n'=360) the coverage of the computed (two-tailed) 95 percent nominal confidence intervals for the test populations falls short but conforms approximately to expectations. More specifically, on the average for the three test populations, 93.2 percent of this particular set of 3000 repeated samples (1000 for each test population), the 95 percent nominal confidence intervals include the value being estimated. Such estimates are, of course, subject to sampling errors. For the next sample size (n=1200, n'=360), the observed average proportion of the 95 percent nominal confidence intervals that include the value being estimated is similar but slightly lower, being about 93 percent. For the two smaller sample sizes (n=880, n'=260 and n=350, n'=160) the proportions are about 92 percent and 91 percent, respectively. While these are statistically significant departures from expectation for normal distributions, the results are nevertheless close enough that the computed confidence intervals can be interpreted as providing useful measures of the precision of estimated error rates, with the observed probabilities being somewhat less than but reasonably close to expectation. They tend to be closer to the nominal probabilities for the larger sample sizes. However, from Table 2-4 it is seen that for the lower tails (below the 2-1/2 percent and 5 percent nominal bounds), or for the upper tails (above the 95 percent and 97-1/2 percent nominal bounds), the probabilities do not tend to be closer to the nominal probabilities for the larger samples. We presume this is because the subsampling ratio n'/n is lower for the larger sample sizes, and especially for the largest sample size used in the analyses.

As seen from Figure 2-2, for the sample sizes in use, the distributions of the estimated overpayment error rates appear to be reasonably close to normal, although still moderately skewed. As discussed in Appendix C, the departure from expected proportions in each of the two tails of the confidence intervals arises because the distributions of payment errors are considerably skewed, resulting in a positive correlation of the estimated standard deviations with the estimated overpayment error rates, and especially because of the wide variability in the estimated standard deviations. As a result, the computed upper and lower nominal 95 percent confidence bounds are both somewhat low.

Figure 2-2. Distribution of estimated payment error rate (based on 1000 samples from Population A)



2.4 An Improved Procedure for Computing Confidence Bounds

The results summarized in Section 2.3 above are for confidence intervals as they are now computed. We have explored several alternatives for computing confidence intervals and describe here an alternative method that involves the use of "Jackknife replicates."[2] The greater the number of Jackknife replicates used, the greater is the precision of the variance estimates, but also the greater the computation costs. Often, in practice, a compromise choice is made and from 30 to 60 replicates are frequently used.

One way that K Jackknife replicates can be formed, after selection of the state and Federal samples for a state, is by first dividing the state sample into K random subsets of equal or nearly equal size (each subset would be a stratified random subsample if the original sample was stratified). A Jackknife replicate is then formed by dropping one of the random subsets from the total sample and retaining in the replicate all of the remaining cases. A total of K overlapping replicate samples is formed by repeating this for each of the K subsets. The Federal findings are used for the cases in a replicate that are members of the Federal subsample.

The regression estimate of the overpayment error rate is made separately for each replicate as well as for the total sample. Then an estimate of the variance of the overpayment error rate for the whole sample is obtained by computing

$$s_{\hat{R}}^2 = \frac{K-1}{K} \sum_{k}^{K} (\hat{R}_k - \hat{R})^2$$

where $\hat{R}_k$ is the estimated overpayment error rate for the k-th Jackknife replicate, and $\hat{R}$ is the estimate for the whole sample.

---

[2]The term "Jackknife" was suggested by John Tukey, a leading statistician, who noted that the method might be used to estimate variances of complex statistics. He noted that the use of Jackknife replicates provides a simple and approximate method for making variance estimates from samples even for complex estimators such as the double sampling regression estimator. He observed that the procedure was a simple but often effective tool, something like using a jackknife as a general-purpose tool.

Another way to form Jackknife replicates starts by defining 2K subsets of the state sample and arranging them into K pairs. The pairs would be random divisions of first-stage sampling units, within strata if the original sample is stratified, or stratified samples within groups of strata of about equal aggregate size. A Jackknife replicate then uses the data in all pairs except one. In that pair, one of the subsets chosen randomly is doubled and the other is omitted. This gives K replicates. Again, the regression estimate is made for each of the replicates. The estimate of the variance is then given by

$$s_{\hat{R}}^2 = \sum_{k}^{K} (\hat{R}_k - \hat{R})^2$$

where $\hat{R}_k$ is the estimated overpayment error rate for the k-th replicate.

With either of the above approaches, confidence bounds can be computed as $\hat{R} \pm t\, s_{\hat{R}}$. With 30 or more replicates, the ordinarily used values of t are t=1.96 for a 95 percent confidence interval and t=1.645 for a 90 percent confidence interval. (If the samples were drawn from normal distributions, these would be appropriate values for t.)

However, in order to reduce the effect of skewness in the distribution of estimated payment error rates, we describe a modification of the above procedure. The modification is to transform the overpayment error rates for each of the K Jackknife replicates and for the total sample by a logarithmic transformation. Such a transformation reduces the skewness of the distribution. If we denote

$$z_k = \log \hat{R}_k$$

$$z = \log \hat{R}$$

then,

$$s_z^2 = \frac{K-1}{K} \sum^{K} (z_k - z)^2$$

if the first described method of forming replicates is used, and

$$s_z^2 = \sum^{K} (z_k - \hat{z})^2$$

if the second method is used.

The lower and upper 95 percent confidence bounds for z are $z_L = z - 1.96s_z$ and $z_U = z + 1.96s_z$.

The lower and upper confidence bounds for $\hat{R}$ are then $\hat{R}_L$ = antilog z_L and $\hat{R}_U$ = antilog z_U.

We have made some tests of this procedure for computing confidence bounds, using 400 repeated independent samples from Population A, for each of four sample sizes used earlier, and for an additional 1500 independent replicates for the largest sample size (n=2400, n'=360) and for an additional 2000 replicates for the smallest sample size (n=350, n'=160). The results are summarized in Table 2-6. (See also Appendix C.)

Table 2-6. Proportion of samples in which the true error rate is above, below, or covered by specified nominal confidence intervals, based on logarithmic transformation of Jackknife replicate estimates, Population A

	Sample size, n/n'				
	2400/ 360	1200/ 360	880/ 260	350/ 160	All sample sizes combined
Number of independent replicates	1900	400	400	2400	5100
Proportion of samples:					
Below .025 point	.017	.032	.028	.023	.022
Below .05 point	.035	.048	.068	.049	.045
Between .05 and .95 points	.890	.890	.867	.889	.888
Above .95 point	.075	.062	.065	.062	.067
Above .975 point	.045	.035	.040	.031	.035

These proportions are considerably closer to the nominal percentages than those observed in Table 2-4 for the confidence intervals as currently computed. Those below the lower 2-1/2 and 5 percent lower confidence bounds, respectively, are reasonably close although they still average somewhat less than the nominal 2-1/2 percent and 5 percent; those above the upper bounds are moderately greater than the nominal 2-1/2 percent and 5 percent. However, the differences, although statistically significant, are small enough to be of relatively minor concern. These results are very encouraging, although some further work is desirable, empirically based on transformations other than the logarithmic transformation, which may reduce the skewness further. Additional details appear in Chapter 3 and in Appendix C.

2.5 Some Further Considerations for Estimating Sampling Error

Current practice in AFDC-QC is to estimate sampling errors (standard errors) of estimated overpayment error rates for a state using only the sample data for the current evaluation period for that state. This is consistent with general practice. However, as indicated earlier, such estimates of sampling errors are subject to large sampling errors, very much larger for a given sample size than in many common sampling situations. As illustrations, Table 2-7 shows estimates of the coefficients of variation of the estimated sampling errors made by current procedures from samples of various sizes drawn from Test Populations A, B, and C. Each coefficient of variation is estimated from 1000 samples drawn independently for each sample size and test population.

The estimated coefficient of variation of $s^2_{\hat{R}}$ is

$$\hat{CV}(s^2_{\hat{R}}) = \frac{\left[\sum_{1}^{1000} (s^2_{\hat{R}i} - \overline{s^2_{\hat{R}}}) \;/\; 1000\right]^{1/2}}{\overline{s^2_{\hat{R}}}}$$

with $\overline{s^2_{\hat{R}}} = \sum_{1}^{1000} s^2_{\hat{R}i} \;/\; 1000$ and i indicating the i-th replicate.

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Table 2-7. Approximate coefficients of variation of $s_{\hat{R}}$ and $s_{\hat{R}}^2$ from 1000 samples drawn from Test Populations A, B, and C for alternate sample sizes,* compared with samples drawn from normal distributions

	Sample sizes			
	n = 2400 n' = 360	n = 1200 n' = 360	n = 880 n' = 260	n = 350 n' = 160
$\hat{CV}(s_{\hat{R}})$				
Population A	.18	.14	.18	.20
Population B	.20	.16	.18	.24
Population C	.27	.22	.26	.30
$\hat{CV}(s_{\hat{R}}^2)$				
Population A	.34	.29	.36	.40
Population B	.40	.32	.37	.46
Population C	.55	.46	.54	.63
For a mean of a simple random sample of n' drawn from a normal distribution				
$\hat{CV}(s_{\bar{x}})$	.037	.037	.044	.056
$\hat{CV}(s_{\bar{x}}^2)$	.075	.075	.088	.112

*The 1000 samples for each sample size from each test population were drawn independently (a simple random sample of n drawn from the test population, and a simple random subsample of n' from the sample of n). The coefficients of variation of $s_{\hat{R}}$ and $s_{\hat{R}}^2$ for a given population and sample size are computed from the same 1000 samples.

Similarly, the estimated coefficient of variation of $s_{\hat{R}}$ is

$$\hat{CV}(s_{\hat{R}}) = \frac{\left[\sum_{i}^{1000} (s_{\hat{R}i} - \bar{s}_{\hat{R}})^2 / 1000\right]^{1/2}}{\bar{s}_{\hat{R}}}$$

with $\bar{s}_{\hat{R}} = \sum_{i}^{1000} s_{\hat{R}i} / 1000$.

The exceedingly skewed distributions of overpayment errors in combination with the use of double sampling and the regression estimator result in these very large sampling errors of estimated variances and standard errors as compared with, for example, the sampling errors of estimates of the variance and standard errors of means based on simple random samples of size n' drawn from a normal distribution[3] (which are also shown in Table 2-7). The large coefficients of variation of the estimated variances and standard errors not only result in relatively large sampling errors for the estimated overpayment error rates, but also cause differences between exact confidence limits (limits that would conform exactly to the nominal probabilities) and the confidence limits as currently computed. As seen earlier (Table 2-4), for the confidence limits as currently computed, the observed coverage probabilities in repeated samples from the test populations differ somewhat from the nominal 95 and 90 percent probabilities, and differ more widely for the upper and lower tails of the confidence intervals considered separately.

[3]See Hansen, M., Hurwitz, W., and Madow, W., ***Sample Survey Methods and Theory***, Vol. 1, (John Wiley & Sons, New York, 1953), pp. 133-148, where theory is given, with illustrations for simple random sampling. The theory and illustrations given there do not cover double sampling with regression estimation, for which the impact of skewed distributions is increased. We note, also, that technically it is not the skewness of a distribution but, rather, its high kurtosis which causes the very large variance of estimated variances. The kurtosis is measured by β = (fourth moment about mean)/σ^4. However, in practice, highly skewed distributions tend to have high kurtosis, and the greater the skewness, the greater the kurtosis. This is strikingly demonstrated in the illustrations in the reference cited. Consequently, we prefer to refer to high skewness in characterizing such distributions, which is readily seen by the eye, rather than high kurtosis, which is not.

A particularly serious problem that results from the large coefficients of variation of $s^2_{\hat{R}}$ is that estimates of the sample size needed to achieve a given level of precision for a state can be subject to wide-ranging error. For example, in a state in which the joint distribution of state and Federal determinations of overpayment error rates corresponds approximately to Test Population C, and with a state sample size of 350 and a Federal subsample size of 160, the coefficient of variation of the estimated variance, $s^2_{\hat{R}}$, would be about 63 percent (and of $s_{\hat{R}}$ about 30 percent).

We examine what might result when the estimated variance for a state is subject to such a large coefficient of variation and is used to determine the sample size needed to achieve a given level of precision. Suppose that an estimate is made for a state of the sample size needed to achieve an estimate of R subject to a standard error of .015. For illustration, we assume that the distribution of overpayment errors in the state is like that of Population C. From the known characteristics of Population C, we compute that if we retain the ratio of sample sizes n'/n = 160/350, a state sample size of n=420 and a Federal subsample size of n=192 would yield such a standard error. However, if one estimated the sample size needed on the basis of $s^2_{\hat{R}}$ estimated from a sample of n'=160 and n=350 (approximately the average annual sample size in use in a number of the smaller states) and if the ratio of n'/n = 160/350 were retained, one would have roughly 1 chance in 20 that the estimates of the Federal and state sample sizes needed would be either as low as n'=38 and n=83 or lower or as high as n'=508 and n=1111 or higher. Such a range is far too wide to provide a useful guide for determining needed sample sizes.

Even for states with large QC sample sizes, the range would be wide. For example, for samples of n'=360 and n=2400 drawn from a state distribution like that of Test Population C, if this ratio of n' to n is retained, there is about 1 chance in 20 that the estimates of needed sample sizes would be as low as n'=38 and n=255 or lower, or as high as n'=305 and n=2036 or higher.[4] Of course, the ratio n'/n might

[4]The needed sample sizes were computed as follows: n' = $S^2_R / \sigma^2_{\hat{R}}$, with $\sigma_{\hat{R}}$ set equal to .015, and

$S^2_R = (\sigma^2_x / \bar{T}^2)[1 - \rho^2(1-n'/n)] = .043$ computed for Population C (see Appendix A) and assuming a fixed

not be retained for such different sample sizes, but the effect of the wide ranging sampling variability would remain. We note that the variance of the estimated variance is somewhat larger for Test Population C, which we have used for illustration, than for the other two test populations.

2.5.1 Pooled Variance Estimates

To reduce the wide sampling variability of the estimated variance of the estimate of R, some consideration has been given by AFDC staff to the use of a pooled estimate of variance in computing the estimated standard error. We regard this as a useful procedure and have developed and evaluated an approach to accomplish this.

We have explored some alternatives that are described in Appendix E. A pooled variance estimation procedure that appears to provide acceptable variance estimates is one in which the states are first ordered on the basis of preliminary pooled unit variance estimates for a prior year or years. We define the preliminary estimated unit variance for state k for this purpose as

$$s_k^2 = \frac{s_{xk}^2}{\bar{t}_k^2}\left\{1 - r_k^2(1-f)\right\}$$

where the symbols are as defined in Chapter 1, with the subscript k added to identify state k.

---

ratio for $n'/n = 160/350$. In practice S_R^2 is unknown and must be estimated from the sample. The estimate of S_R^2 is $n's_{\hat{R}}^2$ as given by Equation (3) in Chapter 1. The observed (not the nominal bounds assuming a normal distribution) 2–1/2 percent and 97–1/2 percent confidence bounds of $s_{\hat{R}}^2$ in 1000 independent replicate samples of n'=160 and n=350, drawn from Population C, and also for n=360 and n=2400 were used to obtain these results.

For this purpose, a uniform value of f=.2 is used for each of the 51 states. A simple mean of such estimated unit variances for the state for two prior years is then computed. The list of states ordered on these average preliminary unit variances is then divided into several relatively homogeneous groups (in Appendix E, we have used 5 groups with 10 or 11 states in each group). For the preliminary unit variance estimates, no use is made of the variance estimates or other sample data for the current year.

The pooled estimates $\tilde{s}^2_{xk}$, $\tilde{t}_k$, and $\tilde{r}_k$ of S^2_{xk}, $\bar{T}_k$, and ρ_k, respectively, are made for state k in a group of m states as follows (with state i different from state k):

$$\tilde{s}^2_{xk} = \left(2n'_k s^2_{xk} + \sum_i^{m-1} n'_i s^2_{xi}\right) / \left(2n'_k + \sum_i^{m-1} n'_i\right)$$

$$\tilde{t}_k = \left(2n'_k \bar{t}_k + \sum_i^{m-1} n'_i \bar{t}_i\right) / \left(2n'_k + \sum_i^{m-1} n'_i\right)$$

$$\tilde{r}_k = \left(2n'_k s_{xyk} + \sum_i^{m-1} n'_i s_{xyi}\right) / \left(2n'_k + \sum_i^{m-1} n'_i\right) \tilde{s}_x \; \tilde{s}_{yk}$$

and $\tilde{s}^2_{yk}$ is defined the same as $\tilde{s}^2_{xk}$, but for the Y variable,

$$s^2_{xi} = \sum^{n'_i} (x_{ij} - \bar{x}_i)^2 / (n'_i - 1)$$

$$s_{xyi} = \sum^{n'_i} (x_{ij} - \bar{x}_i)(y_{ij} - \bar{y}_i) / n'-1$$

$$\bar{t}_i = \sum^{n'_i} t_{ij} / n'_i \, .$$

The symbol x_{ij} denotes the Federal determination of the overpayment error for the j-th case in the Federal subsample in state i, y_{ij} the corresponding state determination of overpayment error, t_{ij} the total payment to case j in state i, and n'_i the size of the Federal subsample for the year in state i.

Note that each of the above pooled estimates is a simple weighted average of the respective state values, with weights equal to the Federal subsample sizes, except that state k, the state for which the pooled unit estimate is being made, is given double weight.

The pooled unit variance estimate for state k is then

$$\tilde{s}_k^2 = \left(\tilde{s}_{xk}^2 / \tilde{t}_k^2\right) \left\{1 - \tilde{r}_k^2 \left(1 - f_k\right)\right\}$$

where $f_k = n'_k/n_k$ is the fraction that the Federal subsample is of the state sample in state k.

The pooled estimate of the variance of $\hat{R}_k$ is then

$$\tilde{s}_{\hat{R}k}^2 = \tilde{s}_k^2 / n'_k \ .$$

This pooled estimate will considerably improve the unit variance estimate for state k, provided that the true and unknown unit variance in each of the other states in the group is not too different from S_k^2, the true (unknown) unit variance for state k. The improvement results because the pooled estimates are made from a much larger sample of cases (about 8 to 14 times as large for an average state) as is s_k^2. Of course, the pooled estimate is, in fact, a biased estimate of S_k^2, the bias depending on how much the expected values of the true state variances and correlations differ from state to state in the group. The analyses and evaluations in Appendix E indicate that very substantial gains result from the use of such a pooled variance estimate for purposes of providing a general measure of precision for a

state. We show in Section 2.5.2 that the pooled variance estimate is not appropriate for use in computing lower confidence bounds, but that the direct state variance estimates are.

We note that this particular pooled unit variance estimator involves very little computational burden. It simply makes use of unit variances and covariances (or correlations) already estimated for purposes of computing direct variance estimates for each state.

It is shown in Appendix E that the simple pooled variance estimates evaluated there have moderately higher correlations across states with the true state variances being estimated than do the direct variance estimates, state by state. At the same time, they have very much smaller variances, by factors of about 6 to 14.

The simple pooled variance described here differs from the one described and evaluated in Appendix E because the one described here obtains weighted averages in which the weight for the specified state is doubled in computing the various terms. From the analyses in Appendix E, we tentatively conclude that this presumably will result in a small increase in the correlation with the true values being estimated, and a small increase in the variance of the composite estimate. The differences should be modest, but some evaluation of this presumption would be desirable.

In summary, because of its much smaller variances, and its moderately higher correlation with the true values being estimated as compared to the direct variance estimates, we conclude that the pooled variance estimator has substantial advantage in providing general precision measures, and in arriving at the expected precision of specified sample sizes. However, it is less useful for computing a lower confidence bound than the direct variance estimate for a state.

2.5.2 Implications for the Choice of Variance Estimators

The results just presented, indicating substantial gains from the use of a pooled variance estimator for a state, might appear to lead to the conclusion that the pooled variance estimator would be superior for all purposes. However, this may not be the case. While the pooled variance estimator achieves substantial gains for most purposes, there remain applications where direct variance estimation, state-by-state, has advantages. We summarize some relevant results in Table 2-8.

The results presented in Table 2-8 are for four different methods of computing confidence intervals. For the "Regular" variance estimator, the confidence bounds are obtained by computing $\hat{R} \pm ts_{\hat{R}}$ where $s_{\hat{R}}$ is the usual direct estimate of the standard error of $\hat{R}$ from the sample for the current year. For the "Jackknife-L", the variance is computed from logarithms of Jackknife replicate estimates, and the confidence bounds are obtained from the inverse transformation of logarithmic confidence bounds, as discussed in Section 2.4 and in Appendix C. For the "Known $\sigma^2_{\hat{R}}$" variance estimator, the variance is not estimated from the sample. Instead, the confidence interval is computed as $\hat{R} \pm t\sigma_{\hat{R}}$, where the parameters of Population A are used in computing $\sigma_{\hat{R}}$ (where $\sigma_{\hat{R}} = S^2_R/n'$ and S^2_R is given in Footnote 4 in Section 2.5). Of course, the parameters for computing $\sigma_{\hat{R}}$ are known for our test population, but would not be known in practice. The results for the unknown true variance are presented to help evaluate the pooled variance estimator. For the pooled variance estimator, the confidence bounds are computed as for the "Regular," except that the pooled estimate of the variance of $\hat{R}$ is used, obtained by procedures discussed in Section 2.5.1, and evaluated in Appendix E.

Table 2-8 shows, in the fourth, fifth, and sixth columns, the estimated mean, standard error, and coefficient of variation (CV) of the lengths of each type of confidence interval. The next two columns show the estimated probability that the true population overpayment error rate is, respectively, to the left and to the right of the computed confidence intervals. The last three columns show the estimated mean, standard error, and coefficient of variation of the lower bounds of the confidence intervals.

Table 2-8. Properties of alternative procedures for computing of confidence intervals for R, for Population A (see text for description)

			Length			Estimated probability		Lower bound		
Sample size	Variance estimator	Confidence level	Mean $\bar{\ell}$	Standard error $\hat{\sigma}_{\ell}$	C.V. $\hat{\sigma}_{\ell}/\bar{\ell}$	R<l.b.	R>u.b.	Mean $\overline{\ell b}$	Standard error $\hat{\sigma}_{\ell b}$	C.V. $\hat{\sigma}_{\ell b}/\overline{\ell b}$
2400/360	Regular	90%	0.0268	0.0053	.20	0.023	0.090	0.0596	0.00653	.11
		95%	0.0319	0.0064	.20	0.009	0.068	0.0571	0.00634	.11
	Jackknife - L	90%	0.0270	0.0054	.20	0.031	0.075	0.0608	0.00660	.11
		95%	0.0322	0.0065	.20	0.017	0.048	0.0587	0.00641	.11
	Known $\sigma^2_{\hat{R}}$	90%	0.0267	0.0000	.00	0.055	0.039	0.0597	0.00798	.13
		95%	0.0318	0.0000	.00	0.027	0.020	0.0571	0.00798	.14
	"Pooled"	90%	0.0267	0.0022	.08	NA	NA	0.0597	0.00790	.13
		95%	0.0318	0.0026	.08	NA	NA	0.0571	0.00780	.14
350/160	Regular	90%	0.0499	0.0105	.21	0.021	0.091	0.0480	0.01153	.24
		95%	0.0595	0.0126	.21	0.006	0.065	0.0432	0.01106	.26
	Jackknife - L	90%	0.0511	0.0111	.22	0.042	0.061	0.0518	0.01169	.22
		95%	0.0614	0.0134	.22	0.019	0.040	0.0486	0.01021	.21
	Known $\sigma^2_{\hat{R}}$	90%	0.0491	0.0000	.00	0.055	0.042	0.0484	0.01488	.31
		95%	0.0584	0.0000	.00	0.028	0.018	0.0437	0.01488	.34
	"Pooled"	90%	0.0491	0.0042	.09	NA	NA	0.0484	0.01460	.30
		95%	0.0584	0.0051	.09	NA	NA	0.0437	0.01430	.29

The first six rows for each sample size in Table 2-8 were obtained by drawing 1000 independent samples from Test Population A. The same 1000 replicate samples were used for computing results for the Regular, Jackknife, and known $\sigma^2_{\hat{R}}$ estimators, for sample size n=2400, n'=360, and another independent set of 1000 replicate samples was used to obtain the corresponding measures for sample size n=350, n'=160.

In the last two rows for each sample size labeled "Pooled", we provide approximate estimates of what would have been obtained had we been able to simulate a pooled variance estimation procedure for a set of states similar to Population A. These results were obtained as explained in Section 2.5.3.

We now examine the implications of the alternative variance estimators for various uses.

For computing confidence bounds after the sample results are available, it appears from Table 2-8, and from Appendix C (as we explain below), that Jackknife-L (i.e., the logarithmic transformation of Jackknife replicate estimates) has advantages over the other alternatives considered, even though the estimated standard error of the length of the confidence interval is about two and a half times greater for this alternative than for the "pooled" variance estimator. Also, the standard error of the lower confidence bound is slightly larger for the Jackknife-L than for the Regular. However, the standard error of the lower confidence bound based on the "pooled" variance estimate is about 20 to 40 percent larger than for lower bounds based on the Regular or Jackknife-L variance estimators. The low standard error of the lower confidence bounds based on both the Regular and Jackknife-L variance estimators arises because of the relatively high correlation of $\hat{R}$ and its estimated standard error (see Appendix C for fuller discussion).

For the "pooled" estimator, the probabilities associated with the tails, that is, beyond the ends of the confidence intervals, are not available. However, the tails for the "known $\sigma^2_{\hat{R}}$" confidence intervals, which use the population parameters instead of sample estimates of $\sigma_{\hat{R}}$, give estimates of those probabilities that are quite

good for the tails. Consequently, because the variances of the estimated standard error for the "pooled" are much smaller than for the "Regular," we assume the tails for the "pooled" might be reasonably close to those for which the known $\sigma_{\hat{R}}$ is used.

We conclude that, in spite of the apparent advantages of the pooled variance estimator for most purposes, the substantially smaller standard error of the lower bound obtained from either the regular procedure or Jackknife-L appears to be sufficiently important as to lead to the choice of one of these procedures for computing the lower bound. Another reason for adopting one of these procedures in computing a lower confidence bound is that each depends only on the estimates from the sample for the current year. One does not have to justify bringing in other data that might be challenged as not completely relevant. The Jackknife-L is preferable to the Regular because the frequencies in the "tails" are considerably closer to the nominal probabilities than are those for the Regular. In summary, we conclude that the Jackknife logarithmic procedure is preferable for computing lower confidence bounds that are to be used for such purposes as the determination of disallowances if they are to be based on lower confidence bounds. In Section 2.4 and Appendix C, we show that it also yields reasonably good results for the upper confidence bounds. The "regular" or current procedure for computing lower confidence bounds may provide acceptable results for less rigorous uses.[5]

The situation is entirely different with regard to estimates of sampling errors for other purposes. At the beginning of Section 2.5.1, we showed great variability of the "Regular" procedure in making estimates of the sample size needed to achieve a given level of sampling error. The range of variability in estimating needed sample sizes will be roughly one-sixth as much or less for the pooled variance estimator as for the direct or for the logarithmic transformation of the Jackknife variance estimator. Similarly, advance estimates of expected sampling errors based on results for prior years will be greatly reduced with the pooled

[5]You have asked for an estimate of the added cost of computing lower confidence bounds by the Jackknife-L procedure as compared with the regular procedure. This cost depends on the computer equipment available and on how the job is programmed. A very rough generous estimate based on the computing equipment we have used for creating the Jackknife replicates and for computing the variances and confidence limits for the test populations is no more than $4,000 for the programming, which is a one-time cost for all states and years, and not more than about $200 for computer time for each state computation.

variance estimator. These advantages are very substantial. Indeed, it appears essential to use a pooled or composite variance estimator in advance variance estimation and in planning needed sample sizes.

Our conclusion is that both approaches have important, but different, uses.

2.5.3 Note on Computation of Characteristics of Confidence Intervals Using the Pooled Variance Estimator

The results presented in Table 2-8 for the "Pooled" variance estimator came only in part from the simulations and were estimated as follows.

The lengths of the confidence intervals for the pooled estimator, ℓ, were assumed to be approximately equal to those for "known $\sigma^2_{\hat{R}}$" since the mean of the pooled estimates of the standard error of $\hat{R}$ should be close to the known $\sigma_{\hat{R}}$. The σ^2_{ℓ} for the pooled estimate was assumed to be equal to one-sixth of the σ^2_{ℓ} for the regular estimator. This is greater than the average value of the ratios of variance of the pooled estimator (with assumed zero bias) to the variance of the regular estimator observed in Appendix E. The mean of the lower bounds, ℓb, for the pooled estimator was assumed equal to the ℓb for known $\sigma^2_{\hat{R}}$ since the intervals would be of approximately equal average length. The estimated standard error of the pooled lower bound, $\sigma_{\ell b}$, follows from the fact that the computed lower confidence bound for the pooled estimator is $\ell b = \hat{R} - ts_{\hat{R}}$. Consequently, the variance of ℓb is

$$\sigma^2_{\ell b} \;=\; \mathrm{Var}(\hat{R}) + t^2\mathrm{Var}(s_{\hat{R}}) - 2t\rho_{\hat{R},s_{\hat{R}}}\sqrt{\mathrm{Var}(R)\mathrm{Var}(s_{\hat{R}})}\,.$$

The $\rho_{\hat{R},s_{\hat{R}}}$ is the correlation of $\hat{R}$ and $s_{\hat{R}}$ and was assumed to be equal to $\sqrt{1/10}$.

This is a rough approximation based on the correlation of x and x+y, where y is the sum of a variable y for a simple random sample of n from a specified population, and x is the sum of a variable x for an independent simple random sample of m, where m/(m+n) equals approximately 1/10. The value 1/10 is chosen because the sample for a particular state in a group may constitute roughly one-tenth of the sample for the entire group. Fortunately, for the approximate relationships that should hold in this case, the $\sigma_{\ell b}$ is not sensitive to any of the terms but the first one, so that the approximations for $\sigma_{\ell b}$ should be reasonab'‧ good.

2.6 Conclusions on the Validity of the Regression Methodology

From the above analyses, supplemented by the fuller analyses presented in later sections and in the appendices, we conclude:

- The regression methodology provides unbiased or at most trivially biased point estimates of the overpayment error rates for the AFDC-QC samples in use.
- The sampling errors estimated from the samples also provide nearly unbiased estimates of the sampling errors of the estimated overpayment error rates. However, they are subject to large sampling errors, much too large to be useful for determining needed sample sizes to yield specified magnitudes of sampling errors.
- A pooled variance estimation procedure is provided that greatly improves estimates of variances, and thus of estimates of needed sample sizes to achieve specified precision levels.
- The confidence intervals as now being computed yield results that, although imperfect, nevertheless provide useful guides to the precision of the point estimates of the overpayment error rates.
- A modified methodology is provided that will yield improved confidence intervals, with closer agreement to the nominal coverage probabilities, especially in the coverage of the tails.

- The point estimates are not affected by imperfections in the confidence intervals as now computed. They provide estimates of the overpayment error rates that are valid within the ranges of error indicated approximately by the computed confidence limits.

CHAPTER 3. CONSIDERATIONS IN CHOICE OF LOWER CONFIDENCE BOUND VERSUS POINT ESTIMATE IN DETERMINING DISALLOWANCES

3.1 Introduction

In this chapter we examine various aspects of the second question we were asked to consider (see Section 1.1), as follows:

- *What are the considerations and constraints involved in the choice of a lower confidence bound versus a point estimate in determining disallowances?*

Disallowances are currently computed and assessed annually for states with estimated overpayment error rates in excess of allowed tolerances. As explained in Chapter 1, the allowed tolerances are specified in legislation. They vary from state to state for years prior to 1984, and are set at 3 percent for 1984 and thereafter. The disallowance for a state is $\hat{D} = (\hat{R} - R_0)A$, provided $\hat{R}$ is greater than R_0, where $\hat{R}$ is the QC regression estimate of R (the true overpayment error rate for the year), R_0 is the corresponding tolerance or target rate (the terms "tolerance" and "target rate" are used interchangeably), and A is the amount of the Federal payment to the state for the year. Under certain circumstances, the disallowance can be suspended or waived by the Secretary of Health and Human Services.

The assessment of disallowances has led to challenges and suits by some of the states, and some have proposed that, because the estimated error rates are subject to sampling errors, a lower confidence bound of R should be substituted for $\hat{R}$ in computing the disallowance. This alternative has also been considered by the Congress. Consequently, it is appropriate to examine and compare the statistical implications of these and other alternatives.

There are important precedents for the use of either the point estimate or a lower confidence bound in various applications of sampling. The choice

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should be guided by the purposes to be accomplished by the assessment of disallowances and is primarily a policy decision, rather than a statistical one, and depends on the goals to be served, as discussed in points (8) through (13) in Section 1.2. However, it has important statistical implications that we will examine in this chapter. We note again, here, that in practice the point estimate is ordinarily and appropriately used where two parties to a funds transfer or payment are involved, and the amount of the payment is determined by a sample estimate. Such applications of samples and the point estimate generally call for samples large enough to yield reasonably precise estimates. Use of a lower confidence bound would result in a disadvantage to one party to the advantage of the other. A lower confidence bound is more likely to be appropriate if the purpose of a sample estimate is to prove carelessness or fraud, such as in auditing, and the consequence may be an assessment of a penalty. In AFDC, the Tax Equity and Fiscal Responsibility Act (TEFRA) of 1982 has been interpreted as requiring use of the point estimate.

When samples are large enough, the difference between the two approaches is reduced, and ultimately, for large enough samples, the difference becomes relatively small. However, the differences are relatively large for the sizes of annual AFDC samples in use. Since large transfers of funds are involved, an understanding of the statistical implications of the alternatives is desirable. We consider this in Section 3.2. We refer to the use of the point estimate in computing annual disallowances as Rule A, and to the use of the lower confidence bound as Rule C. Rule B is a variant of Rule A -- the annual disallowance is based on the point estimate except that the disallowance is waived if the nominal 95 percent lower confidence bound of the error rate is below the tolerance. Rule B will, of course, result in lower disallowances, on the average, than Rule A, because they are waived when the estimated error rate is above, but within likely sampling error range, of the target.

Later (in Section 3.7), we describe still another rule, Rule D. This rule increases the effective sample size for computing disallowances by accumulating the annual disallowances over successive years. The lower confidence bound of the accumulated disallowances is used for computing cash disallowances to be assessed until the sampling error of the total accumulated disallowance is sufficiently small.

The accumulated disallowance based on the point estimates is then used for final settlement.

3.2 Use of Point Estimate Versus Lower Confidence Bound in Computing Annual Disallowances

Table 3-1 illustrates the consequences of using Rules A, B, and C for computing disallowances for alternative values of the excess of the overpayment error rate over the tolerance (column 1), the assumed standard error of the overpayment error rate (column 2), and the size of the Federal payment (column 4). The correct disallowances (computed using the unknown true error rate) for each case are shown in column 5, and the average over all possible samples of disallowances computed with Rules A, B, and C are shown in columns 6, 7, and 8. The coefficients of variation of the disallowances computed with Rule A are shown in column 9. The figures in the table are approximations based on the assumptions stated in the Notes for Table 3-1. The figures in columns 9 through 12 are of principal interest, and apply for any level of the Federal payment to states that have (approximately) one of the seven assumed excess of error rates over tolerance shown in column 1 and one of the two levels of sampling error shown in column 2.

While the figures in columns 9 through 12 of Table 3-1 are approximations, and are not those for any specific states, they are approximately representative of the situation in fiscal year 1984 for many states. For all large states, the sizes of the Federal QC samples are roughly the same, and the state QC samples vary from about 1200 to 2400. The .006 standard error of $\hat{R}$ assumed in Table 3-1 is roughly representative of the average sampling error in 1984 for these states (although the sampling error tends to be somewhat smaller for states with the larger state samples, and somewhat larger for the others). The sampling error of .012 shown in the bottom deck of Table 3-1 is roughly illustrative of a number of medium-sized and smaller states (states with state samples of about 500 to 800).

Column 6 of Table 3-1 illustrates that on the average (over all possible samples) disallowances computed by Rule A are closely equal to the correct

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Table 3-1. Some illustrative approximate average results over repeated samples for annual disallowances computed by Rules A, B, and C*

Excess of overpayment error rate over target $(R-R_0)$ (1)	Standard error of $\hat{R}$ (2)	$\frac{R-R_0}{\sigma_{\hat{R}}}$ (3)	Amount of Federal payment A ($000) (4)	Correct disallowance $D=(R-R_0)A$ ($000) (5)	Average of actual disallowances for Rules A, B, and C: $\bar{D}_A$ ($000) (6)	$\bar{D}_B$ ($000) (7)	$\bar{D}_C$ ($000) (8)	CV of actual disallowances for Rule A $(\sigma_{\bar{D}_A}/\bar{D}_A)$ (9)	Ratio of average actual to correct disallowance: $\bar{D}_A/D$ (10)	$\bar{D}_B/D$ (11)	$\bar{D}_C/D$ (12)
.08	.006	13.3	500,000	40,000	40,000	39,150	35,065	.075	1.00	.98	.88
.05	.006	8.3	500,000	25,000	25,000	24,450	20,065	.12	1.00	.98	.80
.03	.006	5.0	500,000	15,000	15,000	14,700	10,065	.20	1.00	.98	.67
.02	.006	3.3	500,000	10,000	10,000	9,700	5,092	.30	1.00	.97	.51
.01	.006	1.7	500,000	5,000	5,050	3,600	1,084	.57	1.01	.72	.22
.003	.006	.5	500,000	1,500	2,100	550	119	1.07	1.40	.37	.08
.0	.006	0.0	500,000	0	1,200	150	32	1.46	∞	∞	∞
.08	.006	13.3	100,000	8,000	8,000	7,830	7,013	.075	1.00	.98	.88
.05	.006	8.3	100,000	5,000	5,000	4,890	4,013	.12	1.00	.98	.80
.03	.006	5.0	100,000	3,000	3,000	2,940	2,013	.20	1.00	.98	.67
.02	.006	3.3	100,000	2,000	2,000	1,940	1,018	.30	1.00	.97	.51
.01	.006	1.7	100,000	1,000	1,010	720	217	.57	1.01	.72	.22
.003	.006	.5	100,000	300	420	110	24	1.07	1.40	.37	.08
.0	.006	0.0	100,000	0	240	30	6	1.46	∞	∞	∞
.08	.012	6.7	15,000	1,200	1,200	1,175	904	.15	1.00	.98	.75
.05	.012	4.2	15,000	750	750	734	454	.24	1.00	.98	.61
.03	.012	2.5	15,000	450	450	417	168	.40	1.00	.93	.37
.02	.012	1.7	15,000	300	303	214	66	.57	1.01	.71	.22
.01	.012	.8	15,000	150	164	57	16	.88	1.09	.38	.11
.003	.012	.25	15,000	45	96	14	4	1.24	2.13	.31	.09
.0	.012	0.0	15,000	0	72	6	2	1.46	∞	∞	∞

*See *Notes for Table 3-1* for definitions.

Notes for Table 3-1

The rules are defined as follows:

Rule A: $\hat{D}_A = (\hat{R} - R_0)A$ if positive; otherwise $\hat{D}_A = 0$.

Rule B: $\hat{D}_B = (\hat{R} - R_0)A$ if $\hat{R} - 1.645 s_{\hat{R}} > R_0$; otherwise $\hat{D}_B = 0$.

Rule C: $\hat{D}_C = (\hat{R} - 1.645 s_{\hat{R}} - R_0)A$ if positive; otherwise $\hat{D}_C = 0$.

The $\bar{D}_A$ is the average of $\hat{D}_A$, etc.

For each rule, $s_{\hat{R}}$ is the estimate of the standard error of $\hat{R}$ and R_0 is the target error rate. The computations shown in the table depend upon the following assumptions for each model.

For Rule A, the computations assume that $\hat{R}$ is normally distributed and that $\hat{R}$ is an unbiased estimate of the true error rate R.

For Rules B and C, the computations assume that the joint distribution of $\hat{R}$ and $s_{\hat{R}}$ is normal and that they are both unbiased estimates. It is assumed that the correlation of $\hat{R}$ and $s_{\hat{R}}$ is .7 (which is approximately the average correlation observed in simulations for Test Populations A, B, and C (see Appendix C, Table C-1), and that the variance of $s_{\hat{R}}$ is $(\beta-1)\sigma^2_{\hat{R}}/4n'$. We have taken β=4, n'=360 when $\sigma_{\hat{R}}$ = .006, and n'=160 when $\sigma_{\hat{R}}$ =.012. The β=40 is an approximate average value obtained for Test Populations A, B, and C from the assumed relationship

$$\sigma^2_{s^2} = \frac{\beta-1}{n'} \sigma^4$$

and $\sigma^2 = n'\sigma^2_{\hat{R}}$ were each obtained from 1000 replicated independent samples (see Appendix C).

disallowances unless $(R - R_0)/\hat{\sigma}_{\hat{R}}$ is small, say less than about 1.5. It also shows relatively how much disallowances would be overestimated, on the average, when $(R - R_0)/\sigma_{\hat{R}}$ is small. It shows, for example, that if $R - R_0$ is .01 or greater, and if $\sigma_{\hat{R}}$ is approximately .006, the computed disallowance under Rule A will, on the average, be equal or very nearly equal to the correct amount. On the other hand, for a state with $\sigma_{\hat{R}} = .006$, and an excess of the overpayment error rate over the target of only about .003, the average annual disallowance would be 40 percent above the correct disallowance (column 10), and for a state with $\sigma_{\hat{R}} = .012$, the average annual disallowance would be more than twice the correct disallowance.

Rule B is the same as Rule A except that no disallowance is assessed unless there is strong evidence that the true error rate is above the target. More specifically, with Rule B, the disallowance is

$$D_B = \begin{cases} (\hat{R} - R_0)A & \text{if } \hat{R} - t\,s_{\hat{R}} > R \\ 0, & \text{otherwise} \end{cases}$$

with $t = 1.645$ if a nominal 5 percent point (the lower bound of the nominal 90 percent confidence interval) is to be used. Alternatively, a lower confidence bound would be computed using the log-Jackknife-replicate procedure described in Section 2.4, which yields a probability associated with the lower confidence bound that is considerably closer to the nominal probability.

It is seen from Table 3-1 (column 11) that the use of Rule B avoids the overassessment of disallowances that results, on the average, from Rule A when the overpayment error rate is close to the tolerance. Instead, Rule B very slightly underassesses the disallowances, on the average, when $(R - R_0)/\sigma_{\hat{R}}$ is large and, as expected, underassesses them considerably when the sampling error of $\hat{R}$ is large relative to the excess of the overpayment error rate over the target.

We have also evaluated the application of Rule B by using the simulated samples drawn from the Test Populations A, B, and C, and using the

criterion $(\hat{R} - 1.645 s_{\hat{R}}) > 0$ rather than the suggested log-Jackknife-replicate transformation. The results are presented in Table 3-2. We conclude from Table 3-2 that for the three test populations the application of Rule B, using sample estimates of R and $\sigma_{\hat{R}}$, gives quite satisfactory results, i.e., the $\bar{D}_B/A$ in each case is close to $R - R_0$, except for the smallest sample size. For the smallest sample size for Test Population C, especially, the ratio of average computed to correct disallowance (last column) is sufficiently small to result in underestimation of disallowances by about 10 percent. The ratios in the last column of Table 3-2 are reasonably close to and confirm the corresponding approximate ratios in column 11 of Table 3-1, for comparable values of $(R - R_0)/\sigma_{\hat{R}}$. Of course, the results presented in Table 3-2 are averages from 1000 independent replicate samples and are subject to some sampling variability.

The coefficients of variation (CV) of the $\hat{D}_A$ for the illustrative samples are shown in column 9 of Table 3-1. It is seen that the CV increases rapidly as the excess of the overpayment error rate over the tolerance decreases.

For Rule A, the magnitude of the sampling errors relative to the disallowances (illustrated by the "CV of actual disallowances" shown in column 9 of Table 3-1) has been the basis for a concern expressed by some states that the amount of the disallowance may vary widely due to sampling error. This concern has led some of the states to propose the adoption of Rule C for computing disallowances, i.e., that disallowances be computed by using a lower confidence bound instead of the point estimate. The consequences of doing this for a one-tailed 95 percent confidence bound (i.e., a lower 90 percent two-tailed confidence bound) are illustrated in columns 8 and 12 of Table 3-1. If such a lower confidence bound were adopted, the disallowance for a state would rarely exceed the correct value, and then only by a relatively small amount. Also, as seen in Table 3-1, the average of such disallowances would be below, and often far below, the correct disallowance.

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Table 3-2. Average annual disallowances, computed for Rule B, for specified sample sizes (based on 1000 independent samples from each test population, and assuming tolerance of R_0=.03)

Sample size		Standard error of $\hat{R}$: $\sigma_{\hat{R}}$	Standard error of $\hat{R}$: $(R-.03)/\sigma_{\hat{R}}$	Disallowances*: Proportion of samples with $\hat{D}_B>0$	Disallowances*: Average computed disallowance proportion $\bar{D}_B/A$	Disallowances*: Correct disallowance proportion $R-.03$	Disallowances*: Ratio of average computed to correct disallowance
Test Population A (R = .0730)							
2400	360	.0071	6.1	1.000	.0431	.0430	1.00
1200	360	.0079	5.4	1.000	.0424	.0430	.99
880	260	.0093	4.6	0.999	.0426	.0430	.99
350	160	.0129	3.3	0.957	.0422	.0430	.98
Test Population B (R = .0795)							
2400	360	.0071	7.0	1.000	.0489	.0495	.99
1200	360	.0087	5.7	1.000	.0490	.0495	.99
880	260	.0103	4.8	1.000	.0487	.0495	.98
350	160	.0152	3.3	0.984	.0490	.0495	.99
Test Population C (R = .0662)							
2400	360	.0079	4.6	0.997	.0359	.0362	.99
1200	360	.0088	4.1	0.996	.0360	.0362	.99
880	260	.0103	3.5	0.976	.0352	.0362	.97
350	160	.0143	2.5	0.791	.0326	.0362	.90

*See Table 3-1 and Notes for Table 3-1 for definitions.

In Section 3.7 we describe an alternative procedure, Rule D, for computing and assessing disallowances that may have advantages over assessing an annual disallowance solely on either the point estimate or a lower confidence bound. Before doing this, however, we review some of the implications of using a lower confidence bound rather than the point estimate in computing disallowances. These issues include choice of a probability to associate with a lower confidence bound, improved procedures for computing lower confidence bounds, the comparative precision of the lower confidence bounds and the point estimate, a procedure to avoid a concern that poor-quality work on QC in a state could work to the disadvantage of the Federal government by lowering the lower confidence bound, and some limited discussion of optimum sample size considerations.

3.3 Some Implications and Issues Concerning Use of the Lower Confidence Bound

We comment here on a few points that are relevant if the lower confidence bound is to play a role in the computation of disallowances, whether based on Rule B or C discussed above, or on Rule D described later (Section 3.7).

3.3.1 Choice of Nominal Confidence Level

The term "nominal confidence level" refers to the desired probability that a confidence interval include the true value that is being estimated. The actual probability may differ from the nominal, although, with appropriate sample design and sufficient sample size, the actual and nominal probabilities may be reasonably close together. For this discussion, we assume they are equivalent. The issue to be considered is at what level the probability associated with a confidence interval, or with an upper or lower confidence bound, is to be specified.

We assume that a 90 percent confidence interval is defined in such a way that a 5 percent probability is associated with each tail, that is, the lower confidence bound is such that the probability is about 5 percent that it exceeds the value being estimated (which we refer to as the true error rate), and the upper

confidence bound is such that the probability is about 5 percent that it is below the true error rate. Similarly, for a 95 percent confidence interval, the probabilities are about 2-1/2 percent that the lower bound exceeds and are also about 2-1/2 percent that the upper bound is below the true error rate. The higher the specified probability for inclusion of the true value within the confidence interval, the lower is the probability associated with each tail. However, a choice must be made of the confidence level to be used; this is a policy decision.

We note that while practice does and should vary, depending on the circumstances and policy judgments made, in much statistical practice 95 percent confidence intervals are displayed and used as measures of precision. Also, the use of a 95 percent confidence level has been the common practice in computing two-tailed confidence intervals to provide measures of precision in AFDC. While there is no necessary reason for adopting the same probability level for computing a lower one-tailed confidence bound, it seems reasonable and is common practice to do so. In a number of analyses, we have displayed both 90 and 95 percent two-tailed confidence intervals, and corresponding 95 percent (or 5 percent) and 97-1/2 percent (or 2-1/2 percent) lower (and upper) confidence bounds. We have adopted a 95 percent lower confidence bound more generally for illustration (or a 95 percent upper confidence bound in some instances) because it seems to represent the most common practice and is consistent in probability level with the level in use in AFDC for measuring precision. However, to the extent that lower confidence bounds have a role in computing disallowances, the adoption of a confidence level can have a substantial impact on the resulting magnitude of the disallowance, and consequently the choice of an appropriate probability level should be a matter for policy determination.

3.3.2 Improved Procedures for Computing Confidence Bounds

Another issue concerns the way in which the confidence interval, and therefore its lower bound, are computed. The present procedure in AFDC in computing a lower confidence bound, L, is

$$L = \hat{R} - ts_{\hat{R}}$$

using the formulas given by Equations (1) and (3) in Chapter 1, respectively, for estimating $\hat{R}$ and $s_{\hat{R}}$, and using t = 1.96 for a 95 percent confidence interval and for a 97-1/2 (or 2-1/2) percent lower confidence bound. Alternatively, we have suggested above, for consideration, the use of t = 1.645 for a 95 (or 5 percent) percent lower bound. As we have shown earlier (Section 2.3), with the highly skewed distribution of overpayment errors, the probability that the lower bound is greater than the true error rate is much less than the nominal 2-1/2 percent. We have also shown that the results are similar for the lower bound of a 90 percent confidence interval (i.e., for a 95 percent lower confidence bound). In Section 2.4, we have suggested the use of a log-Jackknife replicate method of computing confidence intervals which, on the basis of the analyses we have completed, provides probabilities considerably closer to the nominal levels. As noted before, the results are encouraging, although further work on the problem is desirable, particularly in the search for even more useful transformations.

We also note that the computation of confidence intervals using the log-Jackknife-replicate method involves more computing than if computed by the simpler procedure, but with present computer speeds and costs, the difference seems to be unimportant in relation to the potential impact on disallowances if based on a lower confidence bound (see footnote in Section 2.5.2).

3.3.3 Comparative Precision of Lower Confidence Bound and Point Estimate

In Section 2.5.2 of this report and in Section D.1 of Appendix D we explain why the lower confidence bound of the overpayment error rate has considerably greater precision than the point estimate, contrary to the usual situation. We illustrate the comparisons for three test populations. The principal relevance to this discussion is that possible questions concerning the precision of the lower confidence bound do not mitigate against its use in computing disallowances.

3.3.4 Controlling Impact of Sample Size and of Poor-Quality QC Work on Lower Confidence Bound

Another problem with the use of the lower confidence bound in computing disallowances is that it can be lowered by decreasing the sample size or by lowering the quality of the QC reviews done by the state. The first of these effects can be controlled by insistence on minimum sizes for the state sample and the Federal subsample. Some discussion of the implications of alternative sample sizes appears in this subsection and in Appendix D, and also in Sections 3.4 and 3.5.

It is easier to control sample size than the quality of QC work. The presence of poor quality work can reasonably be suspected by an unusually low correlation between the state and Federal findings for the cases in the Federal subsample. An unusually low correlation, or continued observation of a moderately low correlation (say below .8 or .85) may call for more intensive monitoring of the state's QC operation. The distributions of correlations due to sampling, and the distribution of estimated correlations by states, are given in Appendix D. A study of such distributions, along with updating of such analyses from time to time, can provide insight into correlations that may be lower than can be expected from sampling variability alone.

The impact of low correlations on lower confidence bounds of overpayment error rates can be reduced substantially by adopting a "minimum correlation variance estimator." This is accomplished whenever the estimated correlation in the formula for the variance (See Chapter 1, Equation (3)) is below a specified minimum value, say .8, by replacing the correlation in the formula by the specified minimum value. This decreases the estimated sampling error in such instances, thus increasing the computed lower bound. Such low correlations may occur because of poor-quality QC work, or because of sampling variability. Whichever is the cause, the adoption of the minimum correlation variance estimator provides a reasonable adjustment without having any effect on the point

estimate. The selection and use of such a minimum value is discussed in Appendix D.[1]

3.4 Some General Considerations on Optimum Sample Size

We note first, and strongly emphasize, that, except for a few introductory remarks, this discussion of optimum QC sample size assumes that the *only* role of the QC sample is that of computing disallowances, whereas the principal reason for initiating the QC sample and a principal reason for its continued use is to provide information on the frequency and magnitude of errors and their sources, in order to guide improvement and control of the administration of AFDC. Effectively serving these purposes is an exceedingly important role of AFDC-QC. It is obvious that the payoff through reductions in misspent funds can be very great indeed if overpayment error rates are substantially reduced through such efforts. We note, for example, that the reductions in error rates in recent years (e.g., 1980 through 1984) have been substantial, involving reductions of many millions of dollars in improper overpayment of AFDC benefits.

Presumably, an important part of these reductions has resulted directly and indirectly from QC efforts in the states. Nevertheless, the optimum sample sizes needed for guiding improvements in the design and administration of AFDC are not easily determined. We do not here attempt to make that determination in an objective way, but we do emphasize that the sample, for this purpose, should be large enough to facilitate reasonably precise analyses by population subgroups. These should include important subclasses of recipients, so that the sample would provide separate estimates for those working and not working, those with or without other income sources, and other subgroups, and also for major geographic subdivisions. The latter may help in comparing administrative effectiveness within different operating units within the state units. These types of analyses are

[1]From Appendix D, Table D-2, it is seen that the observed correlations for states have been increasing. The 30th percentile of the estimates of correlations by states increased from .76 in 1981 to .87 in 1984. From these it seems that, until additional evidence is available, the choice of a minimum r of .80 to .85 would be quite reasonable. Presumably, the lower values of the estimated correlations by states in the table reflect to a considerable degree the consequences of sampling variability (see Figures D-2A, B, and C).

important and necessary, but it is not easy to specify the sample size needed for such analyses. These analyses are to be done primarily with the state samples which are, of course, considerably larger than the Federal subsamples. For analyses by various subclasses, it may be useful to accumulate samples over two or three years, and also to plot control charts for subclasses based on quarterly or more frequent QC results. The role of the Federal subsamples in this regard is simply to monitor the state QC efforts so that the state samples will be reasonably effective in identifying sources of errors by type.

One of the important considerations concerning the sample sizes that are needed to provide information for corrective action (and also for computing disallowances) is that when a state welfare system is "under control," that is, it has reduced its overpayment error rate in total and in the major jurisdictions or subclasses to an acceptably low level, perhaps to or below the current three percent tolerance, there may be little to gain from additional efforts at corrective action (and nothing to gain from disallowances). Consequently, it seems reasonable for such a state to reduce the QC program to a monitoring role, primarily to provide assurance that the overpayment error rate does not rise substantially again. This could be done with relatively small sizes of state and Federal samples (for example, perhaps 300 to 600 for the state sample and 150 for the Federal subsample).

We mention one other consideration with regard to sample size: any effort to optimize sample size through a cost-benefit approach must take account of the total expenditures involved. The exception is the case mentioned in the preceding paragraph, where the administration of AFDC is demonstrably under good control.

From a cost-benefit point of view, it may be worth using only a relatively small QC sample in the smaller states. Cost-benefit considerations call for higher precision and greater detail for large states. Large samples can provide analyses at shorter time intervals, or by major administrative areas, or for population subgroups, and may greatly facilitate identifying problems and taking corrective action. In New York, for example, in fiscal year 1984 the cost of AFDC was $957 million, while in Wyoming it was about $6 million, or about 6/10 of one percent of the New York cost. Delaying or failing to take effective corrective action

in Wyoming could not noticeably impact total erroneous expenditures in the AFDC program, whereas delay or ineffective action could be enormously costly in New York (and in each of a number of other large states). It would be totally cost-ineffective to call for equal sample sizes or equal precision in these two states -- too costly to take a large sample in Wyoming, and large losses would be risked if a small sample were used in New York, at least until the error rate is acceptably low.

We think the need for larger samples in the larger states is reasonably obvious from a cost-benefit point of view without further comment and justification. The analysis in Section 3.5 of optimum sample size for determining disallowances using a lower confidence bound provides a rather striking illustration of this point.

3.5 Optimum Sample Size for Computing Disallowances

We now turn to consideration of optimum sample size when the sole purpose of QC is assumed to be the computation of disallowances, and the goal is to minimize the overall cost to the Federal government of overpayment errors in the AFDC program, taking joint account of the cost to the Federal government of QC and of the returns from disallowances.

When the point estimate is used to compute disallowances (Rule A) it is not feasible to determine objectively an optimum sample size based on expected (or average) results. This is because, whatever the sample size, the sampling errors of the estimates of the overpayment error rates are both positive and negative, and when the estimated error rate is used in the computation of the disallowance, the long-run average effect of the sampling error in the estimation of disallowance is close to zero for high error rates and is a decreasing function of the sample size. If the true error rate is close to the target rate, the average of the disallowances is positive (as discussed earlier), and increasingly so, as the sample size is decreased. Consequently, it is no longer true that there is an approximately equal chance of positive and negative errors. However, it is still true that the Federal government gains more, on the average, as the sample size is decreased, since the average expected disallowance is larger. (See Appendix F.)

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Thus, from a simplistic point of view, if the point estimate is used, the optimum sample size is to make the state sample and the Federal subsample as small as possible (like a sample of 2), and still make it possible to make an estimate. Of course, this is ridiculously small; neither the Federal government nor the state would be willing to deal with such a ridiculously small sample. It just means that we do not have a basis for obtaining an optimum sample size based jointly on cost and expected or average return from disallowance.

One might make some assumptions about the cost of errors in the point estimate that result in much too large a disallowance in some years, and much too small in others, and possibly arrive at an optimum based on the costs and disadvantages of such variability. We have not taken this approach here, because it does not appear very promising, at least at the present stage of this analysis. We conclude that the determination of optimum sample size for computing disallowances by Rule A is a judgment decision, not effectively guided by a mathematical solution, at least for the present.

The situation would be quite different if the lower confidence bound were to be used in computing disallowances. In this case, from the Federal point of view, the larger the samples for a state, the smaller the sampling error, and therefore the higher the average disallowance. But to achieve a larger sample costs additional Federal funds, both for the Federal subsample and for the state sample. Under these circumstances, it is possible to determine the sample size that maximizes the Federal return. This is done in Appendix G where details are presented. We summarize some results here.

In this analysis it is assumed that the Federal costs for QC include half of the cost of the state QC sample, and the full cost of the Federal QC sample. We used, for determining unit costs, the costs and caseloads quoted in a memorandum from OFA outlining a meeting on September 4, 1984, with the Ways and Means Staff regarding the AFDC Quality Control System and Error Rate Disallowances.[2]

[2]Memorandum to Debbie Chassman from Barbara Levering, Department of Health and Human Services, Office of Family Assistance, Social Security Administration, dated August 31, 1984, *September 4 Meeting with Ways and Means Staff on AFDC Quality Control System and Error Rate Disallowances* and attached outline on *Briefing Points for Ways and Means Staff.*

The resulting assumed unit costs were $130 Federal cost (1/2 total unit cost) of the state sample per case, and $330 per case for the Federal subsample. We also assumed a target error rate of 3 percent, as called for in 1984 and afterwards by present legislation. Various levels of total Federal payments were assumed that are illustrative of payment levels in the various states. We also assumed that the Federal subsample size was 15 percent of the state sample size, as it is in some of the larger states. The computations could readily be carried through for other subsampling fractions, and would yield similar results. We also assumed three levels of the standard deviation of the payment errors, that the correlation of state and Federal findings was .9, and that the correlation of $\hat{R}$ and $s_{\hat{R}}$ was .8.[3] Given the above assumptions, we obtained the summary results displayed in Table 3-3.

Table 3-3. Approximate optimum Federal sample sizes (n') for computing annual disallowances based on a lower confidence bound (Rule C), for alternative levels of total Federal payment, and of excess of overpayment error rate over the target rate

Size of Federal payment ($1,000,000)	Standard deviation of payment errors	Excess of payment error rate over target				
		.01	.02	.03	.04	.06
20	30	--	--	84	84	84
	50	--	--	117	117	117
	70	--	--	140	147	147
50	30	--	154	154	154	154
	50	--	215	217	217	217
	70	--	239	271	271	271
300	30	510	510	510	510	510
	50	673	716	716	716	716
	70	545	800+	800+	800+	800+
500	30	716	716	716	716	716
	50	800+	800+	800+	800+	800+
	70	800+	800+	800+	800+	800+

[3]Elsewhere we have assumed .7 for this correlation (see, for example, Appendix E). This .8 assumption here was based on early results. We have not regarded it as worthwhile to recompute assuming a correlation of .7.

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We note that the optimum Federal sample size becomes zero (denoted by "--" in the table) as the excess of the overpayment error rate over the target gets small. This means that, in such instances, the amount recovered in disallowance is equal to or less than the Federal cost of QC sampling. On the other hand, the optimum sample sizes increase and become considerably larger than the present Federal subsample sizes as the excess of the overpayment error rates over the target increases, and as the total Federal payment becomes large. (Note that an entry of 800+ in the table signifies that the optimum Federal sample size is greater than 800. Our computation did not extend beyond that size.) We emphasize, again, that this optimization is for separate computation of disallowances each year, using the lower confidence bound in the computations (Rule C), and that the optima are computed only to maximize net return from disallowances to the Federal government.

From the point of view of a state (instead of the Federal government), the effect of jointly minimizing a state's cost of conducting the QC operation and its losses from disallowances is totally different. Obviously, if a lower confidence bound is used to compute disallowances, the optimum size of a state sample is the smallest that it is permitted to use, for this would increase the sampling error and therefore lower the lower confidence bound and the disallowance. It would simultaneously reduce the cost of QC.

3.6 The Impact in FY 1981 of Three Disallowance Rules -- Rules A, B, and C

For fiscal year 1981, disallowances were assessed against 27 states and Puerto Rico (see Table 3-4). Waivers were granted in six of those cases. The disallowances assessed were computed by Rule A, that is,

$$\hat{D} = (\hat{R} - R_0)\,A, \text{ if positive,}$$

where R_0 and A vary from state to state. (For the states of Arizona and Texas, a somewhat different and more complex computation was used, but the difference is not relevant to this discussion.)

Table 3-4 presents the assessed disallowances for Rule A. It also presents, for comparison, what they would have been if computed by Rules B or C (as described in Section 3.2). Rule B computes the disallowances as in Rule A, except that if the 95 percent lower confidence bound is less than the target level, the disallowance is waived. The lower confidence bound is computed as $\hat{R} - 1.645 s_{\hat{R}}$ where $\hat{R}$ and $s_{\hat{R}}$ are computed by the current procedures (Equations (1) and (3) in Chapter 1 except for states that use a stratified sampling estimator).

Rule C bases the disallowance on the lower bound alone, as has been suggested by some. That is, the disallowance is computed as the excess of the lower bound over the target rate, applied to the Federal payment:

$$\hat{D} = (\hat{R} - 1.645 s_{\hat{R}} - R_0)\, A, \text{ if positive.}$$

The totals for all 27 states are shown for each rule, as well as the totals reduced by the amounts for the states for which the disallowance was waived. Thus, after waivers, the total disallowance is 17 percent less for Rule B than for Rule A, and is 58 percent less for Rule C than for Rule A. The larger aggregate loss for Rule C occurs because sampling errors are large enough that the 95 percent lower confidence bounds are considerably below the point estimates.

3.7 An Alternative Rule for Computing Disallowance -- Rule D

We describe here another rule, designated Rule D, which combines certain attractive characteristics of Rules A and C, but mitigates certain unattractive characteristics from the points of view of the Federal government and of the state.

Disallowances as now computed by Rule A are subject to relatively large sampling errors in many states, even with the larger annual samples in use in the QC program in some states. These relatively large sampling errors can lead to substantially overstated or understated annual disallowances in a given year.

Table 3-4. Disallowances based on alternative rules, FY 1981

State	Federal expenditure	Disallowance		
		Rule A	Rule B	Rule C
AL	55,257,339	46,527	0	0
AZ	18,204,168	209,475*	293,014*	1,642*
CA	1,270,296,772	35,066,542	35,066,542	17,449,396
CO	47,081,958	1,898,203	1,898,203	1,104,828
CT	102,601,922	313,038	0	0
FL	121,842,954	3,467,041	3,467,041	2,408,721
HI	46,619,415	1,211,639	1,211,639	283,859
ID	14,481,785	691,187	691,187	243,773
IN	83,266,989	112,744	0	0
KS	47,251,492	1,902,865	1,902,865	1,174,489
MD	113,146,541	1,325,172*	0	0
ME	40,439,640	167,744	0	0
MN	134,920,297	571,253	0	0
NE	27,006,307	279,947	0	0
NJ	270,515,844	1,279,810*	0	0
NM	32,394,291	2,553,545	2,553,545	1,800,804
NY	755,115,221	6,269,722	0	0
OH	333,931,792	3,930,043	0	0
OK	58,315,715	1,508,394	1,508,394	526,570
SC	56,158,502	1,003,946*	1,003,946*	456,559*
SD	11,866,284	12,804	0	0
TN	59,079,920	1,754,496	1,754,496	1,093,902
TX	87,575,396	1,112,295	1,396,127	273,375
UT	34,319,580	299,747*	0	0
VT	26,751,544	225,194*	0	0
WA	118,607,888	4,161,714	4,161,714	1,750,039
WY	4,235,182	412,782	412,782	324,958
Totals	3,971,284,738	71,787,869	57,321,495	28,892,915
Total, after waivers		67,444,525	56,024,535	28,434,713

*Denotes that the disallowance was waived.

Rule A: The current rule, based on the point estimate.
Rule B: Based on excess of point estimate over the target error rate, but only if the 95 percent lower confidence bound is above the target error rate.
Rule C: Based on the excess of the 95 percent lower confidence bound over the target error rate.

Note: A somewhat different computation of the disallowance was done for the states AZ and TX than would result from a simple application of Rule A. The figures for these states in the column headed "Rule A" reflect the disallowance as assessed rather than the disallowance computed by Rule A. On the other hand, the figures in the column headed "Rule B" are computed by Rule B, which for these states gives the same disallowance as obtained by Rule A.

The relative magnitude of these sampling errors is illustrated by the coefficients of variation shown in column 9 of Table 3-1. The limits of 95 percent confidence intervals would vary from sample to sample, but, on the average, would correspond to about two times the coefficients of variation shown in that table. For example, the standard error of $\hat{R}$ of .006 shown in column 2 is approximately illustrative of the standard errors in the states with the larger QC samples (a state sample of about 2400 and a Federal subsample of about 360). Column 9 shows that for such a large state, with an error rate of 5 percent (i.e., $R - R_0 = .02$, with a target level of $R_0 = .030$, and a sampling error of .006) the coefficient of variation of the estimated disallowance is .30. Consequently, for such a state, the bounds of the 95 percent nominal confidence intervals would average between 60 percent above and 60 percent below the correct disallowance. About 5 percent of the time, the value being estimated will be either below or above the confidence interval. For a smaller state with a sampling error of .012, this range would be approximately doubled. These are relatively wide ranges due to sampling error. As seen from the table, they would be much larger for states with the same sampling errors, but with overpayment error rates closer to the 3 percent target, and of course would be considerably smaller for states with overpayment error rates considerably above the illustrated rate of 5 percent.

From the point of view of the states, the problem of the large overestimates of disallowances that will occur in some years would be avoided by use of the lower confidence bound (i.e., Rule C) instead of the point estimate. However, as illustrated in column 12 of Table 3-1, and also in Table 3-4, with present annual sample sizes this would result in large losses to the Federal government by consistently and substantially understating the disallowances that would be assessed if the true payment error rates were known.

Another problem with Rule A is that disallowances are assessed only when the estimated error rate is above the target. Thus, because of sampling variation, a state may be assessed a disallowance when in fact the payment error rate is equal to or below the target rate. Moreover, since negative disallowances are not permitted by Rule A, such disallowances would not be compensated for over time. Consequently, a state that is at or near the target rate, above or below, would on the average be improperly assessed disallowances. A state whose error rate is

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moderately above the target rate would, on the average, be assessed a considerably larger disallowance than it would be if the true error rate were known.

To eliminate or substantially reduce these problems, we have developed and have simulated the application of Rule D for computing disallowances. This rule accumulates the full disallowances across years, computed by Rule A except that negative total disallowances are allowed to accumulate on the books. It assesses an annual *cash disallowance* on the basis of a lower confidence bound of the accumulated total disallowance. The final accumulated settlement is based on the accumulated disallowance based on the point estimates and is made when the relative sampling error (the coefficient of variation) of the accumulated total disallowance is acceptably small, say less than 10 to 15 percent. What is acceptably small is a policy decision.

Convenient computation formulas are given in Appendix H. Over a few years, the application of Rule D greatly increases the effective sample size and greatly reduces the large annual fluctuations of disallowances due to sampling errors. Prior to a final settlement date, at which time the accumulated disallowance is based on the annual point estimates and a much larger sample, the Federal government recovers somewhat less in cash but avoids considerably overassessing some states each year.

We note that under this procedure, the lower confidence bound of the accumulated disallowance estimate for a given year, say year i, may be less than the lower confidence bound of the accumulated disallowance in the prior year, i–1. In this event, the Federal government could pay the difference to the state. The total accumulated disallowance would then remain the accumulation of the annual disallowances. Alternatively, credit could be given against future disallowances. The choice is a policy decision.

We note, also, that when the excess of the true error rate over the tolerance becomes small, say, less than one percent, the coefficient of variation of the accumulated disallowance remains large (above 10 or 15 percent) for many years, and a settlement would be long delayed. This is as it should be, because the amount of settlement in such an instance cannot be estimated acceptably from a sample of

any reasonable size, and therefore even after the sample is accumulated over a number of years. We also note, as will be seen later, that under Rule D, for states for which the sample is large and the excess of the overpayment error rate over the tolerance is also large, a cash settlement may be reached within two or three years or even annually.

While the application of Rule D will result in considerable reduction initially in the cash withholding by the Federal government, a temporary cash loss may be acceptable for a few years in order to avoid substantially overassessing some states in individual years. Interest charges (or payments) might be introduced for the amounts carried on the books, in which event the disadvantage to the Federal government would appear to be reduced or removed. On a relative basis, the accumulated disallowance based on the lower confidence bound would approach, over a number of years, the full disallowance based on the point estimate.

Table 3-5 illustrates the expected (average) consequences of applying Rule D to a state with an annual sampling error of .006, and also of .012, for a fixed annual Federal payment of $100 million. It shows, for varying levels of the true error rate, the expected accumulated disallowances over a period of 1 to 16 years, based on Rule D, compared with those for Rules A and C. Appendix H describes the application of Rule D more fully, and it contains 16 illustrative examples of disallowances computed by Rules D and A, for successive years. The tables display random variations as they may occur in practice, for various values of the true overpayment error rate, and of the standard error of the estimates.

It is seen from Table 3-5, and from Appendix H, that Rule D provides a compromise approach between Rule C and Rule A. In the first year, with Rule D, the cash disallowances are the same as for Rule C, although the balance of the full Rule A disallowance is recorded as an obligation available for offset in subsequent years.

While the accumulations are carried through 16 years in Table 3-5, they could be cut off after the estimated coefficient of variation becomes acceptably small and the accumulation process would begin again. The accumulated settlement

Table 3-5. Expected accumulated disallowance compar[illegible] for Rules A, C, and D

		Accumulated measures								
					Expected disallowance					
							Rule D			
$R - R_0$	Year	Federal payment ($1 mil.)	Standard error of $\hat{R}$	Correct dis-allowance	Rule A	Rule C	Cash	Book	Total	CV of total
.05	1	100	.0060	5.0	5.0	4.0	4.0	1.0	5.0	.120
	2	200	.0042	10.0	10.0	8.0	8.6	1.4	10.0	.085
	4	400	.0030	20.0	20.0	16.1	18.0	2.0	20.0	.060
	8	800	.0021	40.0	40.0	32.1	37.2	2.8	40.0	.042
	12	1,200	.0017	60.0	60.0	48.2	56.6	3.4	60.0	.035
	16	1,600	.0015	80.0	80.0	64.2	76.1	3.9	80.0	.030
.05	1	100	.0120	5.0	5.0	3.0	3.0	2.0	5.0	.240
	2	200	.0085	10.0	10.0	6.1	7.2	2.8	10.0	.170
	4	400	.0060	20.0	20.0	12.1	16.1	3.9	20.0	.120
	8	800	.0042	40.0	40.0	24.2	34.4	5.6	40.0	.085
	12	1,200	.0035	60.0	60.0	36.3	53.2	6.8	60.0	.069
	16	1,600	.0030	80.0	80.0	48.4	72.1	7.9	80.0	.060
.03	1	100	.0060	3.0	3.0	2.0	2.0	1.0	3.0	.200
	2	200	.0042	6.0	6.0	4.0	4.6	1.4	6.0	.141
	4	400	.0030	12.0	12.0	8.1	10.0	2.0	12.0	.100
	8	800	.0021	24.0	24.0	16.1	21.2	2.8	24.0	.071
	12	1,200	.0017	36.0	36.0	24.2	32.6	3.4	36.0	.058
	16	1,600	.0015	48.0	48.0	32.2	44.1	3.9	48.0	.050
.03	1	100	.0120	3.0	3.0	1.1	1.1	1.9	3.0	.397
	2	200	.0085	6.0	6.0	2.2	3.2	2.8	6.0	.283
	4	400	.0060	12.0	12.0	4.5	8.1	3.9	12.0	.200
	8	800	.0042	24.0	24.0	9.0	18.4	5.6	24.0	.141
	12	1,200	.0035	36.0	36.0	13.5	29.2	6.8	36.0	.115
	16	1,600	.0030	48.0	48.0	18.0	40.1	7.9	48.0	.100
.01	1	100	.0060	1.0	1.0	0.2	0.2	0.8	1.0	.568
	2	200	.0042	2.0	2.0	0.4	0.7	1.3	2.0	.420
	4	400	.0030	4.0	4.0	0.9	2.0	2.0	4.0	.300
	8	800	.0021	8.0	8.1	1.7	5.2	2.8	8.0	.212
	12	1,200	.0017	12.0	12.1	2.6	8.6	3.4	12.0	.173
	16	1,600	.0015	16.0	16.2	3.5	12.1	3.9	16.0	.150
.01	1	100	.0120	1.0	1.1	0.1	0.1	1.0	1.1	.878
	2	200	.0085	2.0	2.3	0.2	0.3	2.0	2.3	.652
	4	400	.0060	4.0	4.6	0.4	0.9	3.3	4.2	.536
	8	800	.0042	8.0	9.1	0.9	2.8	5.2	8.0	.420
	12	1,200	.0035	12.0	13.7	1.3	5.3	6.7	12.0	.346
	16	1,600	.0030	16.0	18.2	1.7	8.1	7.9	16.0	.300

would then be based on the accumulated results of the annual point estimates, and on a sample several times larger than the sample for a single year. The cut-off time would be extended more or less indefinitely for states with overpayment error rates near the target. Various modifications of Rule D could also be considered.

An important consequence of applying Rule D is that, prior to final settlement, the accumulated cash disallowance and thus the cash disallowance assessed in each individual year is determined from a confidence interval computed from the much larger accumulated QC sample. At the time of final assessment of the full disallowances the samples are much larger than the annual samples. Such an approach substantially reduces the wide variability in annual disallowances that occurs due to sampling variability under present procedures, especially for states with error rates close to the target or with small samples. This wide variability is illustrated, in detail, in the column headed "AFDC" of Tables H-1 through H-16 in Appendix H, giving the annual cash disallowance that would be assessed under the present rule. (Note that negative values in this column would, under present rules, result in a zero disallowance.)

Another consequence of Rule D is that it allows only a very low probability of assessing any cash disallowances against a state that is, in fact, meeting or near (above or below) the target payment error rate but which would often be assessed disallowances under the present procedure, due to sampling variability.

We note that in the application of Rule D, there may be an unusually large Federal withholding in the year of a final settlement. If desired, this adjustment to the point estimate could be spread over two or three years to make a smoother series of disallowances.

A question that arises is how to treat waivers in the application of Rule D. Waivers occur when, for various reasons, all or a part of the disallowance that would otherwise be assessed against a state for a particular year is waived. In Table 3-4 above, full waivers for 1981 were granted for six states. No specific question arose because all waivers were full waivers. With Rule D, as with the other procedures, the disallowance after a full waiver would be zero. The added accumulation for that year would then be zero. For a partial waiver, the nonwaived

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part of the disallowance would be accumulated. The computation of the estimated standard error would reflect appropriately whatever waiver was allowed.

In Table 3-9, we illustrate computation of disallowances for each state by Rule D for the four fiscal years 1981 through 1984, the years for which information is currently available. Since waivers are available only for 1981, we have made the computations without waivers.

We note that, because of some exceedingly high target rates for some states for 1981 (and to some extent for 1982, also) the results presented in Table 3-9 provide a quite distorted picture from the application of Rule D. For example, Illinois has a target rate for 1981 of 12.7 percent. Its observed rate of 8.3 percent is still a high error rate. If Rule D were to be applied to Illinois beginning in 1981, the state would receive an initial book credit of 17.5 million dollars, to be credited against future disallowances. It seems highly undesirable to initiate Rule D for such a state, and more appropriate to initiate the rule for a state with a negative disallowance only if the target for the state is below a specified level, for example, below 8 percent. Of course, the setting of this specific target level is a policy determination. If the specified target level for 1981 were set at 8 percent, then, of the 17 states with 1981 target rates over 8 percent, only one (Maryland) with a 1981 target rate of more than 8 percent has a 1981 observed overpayment rate above its target rate.

In Table 3-6, we provide a summary of the aggregate results from the application of Rule D for two levels of the allowable 1981 target rate (8 percent and 10 percent) for the initiation of Rule D, assuming that the application of Rule D begins in 1981. Excluded from these respective summaries are the 16 states with 1981 target levels above 8 percent for which the computed disallowances are negative, and the 6 states with 1981 target levels above 10 percent for which the computed disallowances are negative (see Table 3-9).

Table 3-6. Summary of aggregate disallowances from application of Rule D to eligible states,* 1981-1984 (thousands of dollars)

		Annual (Rule A)		Accumulated (Rule D)		
		Annual (000)	Cumulated (000)	Cash (000)	Book (000)	Total (000)
Allowable target rate in 1981 is 8 percent or less						
Total	1981	70,837	70,837	28,901	34,542	63,443
	1982	88,137	158,974	81,422	63,576	144,999
	1983	119,836	278,810	179,908	79,407	259,315
	1984	158,750	437,560	313,796	102,723	416,518
Allowable target rate in 1981 is 10 percent or less						
Total	1981	70,837	70,837	28,901	18,941	47,842
	1982	88,518	159,355	81,422	41,268	122,691
	1983	124,755	284,110	179,908	60,421	240,329
	1984	173,591	457,701	320,846	91,092	411,938

*Eligible states are those that have 1981 target overpayment rates that are less than the allowable target, or that exceed the allowable target but have a positive disallowance for 1981.

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Table 3-7 provides a summary of the additional disallowances that would be assessed for those states that would reach a full settlement some time during the four-year period for which data are available if an estimated 15 percent coefficient of variation were the criterion for settlement on the basis of the point estimate. Table 3-8 gives similar results if the criterion for a full settlement were an estimated coefficient of variation of 10 percent.

The District of Columbia is not included in the summaries provided in Tables 3-7 and 3-8 because its target rate was 16.3 percent in 1983 with a negative computed disallowance. For D.C., Rule D would have been initiated in 1982 because the disallowance was then positive, and presumably a complete settlement would have been made for D.C. for each of the years 1982, 1983, and 1984 since its cv in each of these years was less than 10 percent. The total settlement for the three years would have been $9,743 thousand.

Table 3-7. States reaching full settlement by or before 1984, if Rule D were initiated in 1981, and if a 15 percent estimated cv were adopted as the criterion

	Full settlement at end of fiscal year		Added settlement		
				Percent of Federal payment	
State	Year	cv	Amount ($000)	This year	Cumulative
Arizona	1983	.14	935	2.4	1.2
Colorado	1984	.15	1,207	2.3	0.6
Florida	1984	.15	2,364	1.6	0.5
Michigan	1983	.15	9,961	1.8	0.6
(Mich.)	1984	.12	1,658	0.3	0.3
New Mexico	1982	.13	935	3.0	1.5
New York	1983	.15	18,177	2.1	0.7
S. Carolina	1983	.14	1,107	2.1	0.7
(S.C.)	1984	.11	100	0.2	0.2
Wyoming	1981	.13	88	2.1	2.1
Total			36,532		

Table 3-8. States reaching full settlement by or before 1984 if Rule D were initiated in 1981, and if a 10 percent estimated cv were adopted as the criterion

	Full settlement at end of fiscal year		Added settlement		
				Percent of Federal payment	
State	Year	cv	Amount ($000)	This year	Cumulative
Michigan	1984	.10	11,619	1.9	0.5
S. Carolina	1984	.10	1,207	2.2	0.6
Total			12,826		

In summary, assuming the 8 percent 1981 target level, the total cash disallowance would be:

	Amount ($000)	Percent of total
Accumulated total cash, 1981 through 1984, from Table 3-6	$ 313,796	73.6
Add cash from 10 complete settlements (Table 3-7)	36,532	8.6
Add cash from complete settlements for D.C. (not included in Table 3-6)	9,743	2.3
Total cash disallowances assessed over the four years	360,071	84.5
Total accumulated on the book at the end of the four years (102,723 from Table 3-6, less the additional 36,532 from complete cash settlements)	66,191	15.5
Total accumulated disallowances in four years, cash plus book	426,262	100.0

Due to possible minor differences from rounding, and especially because waivers are not available and used in the results presented, and perhaps because of other factors, Tables 3-6 through 3-9 may differ somewhat from the final

determinations if Rule D were to be applied. Nevertheless, they provide satisfactory illustrations of the kinds of results that would occur from applying Rule D.

3.8 Summary

A primary purpose of the quality control program in AFDC is to measure the error rates and to identify likely causes of high rates, in order to guide corrective action. Another major purpose is the assessment of disallowances, based on QC estimates of overpayment error rates, in order to recover Federal funds that have been paid because of overpayment errors above target levels, as prescribed by law. The assessment of disallowances may also be an important factor in influencing states to improve their administration and procedures, and thus to reduce their error rates. The disallowances are currently computed annually using point estimates. A number of states have presented arguments for the use of lower confidence bounds in the assessment of disallowances because of the impact of sampling errors on the assessments. The statistical consequences of using the lower confidence bound versus the point estimate have been examined, and some alternative procedures for computing disallowance have been described. They make use of the point estimate, the lower confidence bound, or both, and one procedure accumulates the computations of disallowances over time in order to reduce the effect on the annual disallowance of large sampling errors. The statistical implications of the four alternatives have been examined in detail and illustrated with examples.

Table 3-9. Application of Rule D to states

		Annual Statistics				Rule A Disallowance *		RULE D								
								Annual Values								
STATE	Year	Fed Contrib	Target	R-hat	s.e.	Annual (if positive)	Cumulated	Cash	Book	Fed Contrib	R-hat	sigma(D)	Cash	Book	Total	cv
AK	1981	17,163,771	.221241	0.18189	0.02625	-675,412	0	0	-675,412	17,163,771	0.1819	450,549	0	-675,412	-675,412	0.67
	1982	16,140,020	.130621	0.12086	0.01871	-157,543	0	0	-157,543	33,303,791	0.1523	542,389	0	-832,954	-832,954	0.65
	1983	15,019,616	.040000	0.15496	0.02126	1,726,655	1,726,655	0	1,726,655	48,323,407	0.1531	629,404	0	893,701	893,701	0.70
	1984	18,670,392	.030000	0.06827	0.01388	714,516	2,441,171	488,522	225,994	66,993,799	0.1295	680,666	488,522	1,119,695	1,608,217	0.42
AL	1981	55,257,339	.076399	0.07724	0.00816	46,471	46,471	0	46,471	55,257,339	0.0772	450,900	0	46,471	46,471	>1.00
	1982	51,190,010	.058200	0.05293	0.00682	-269,771	46,471	0	-269,771	106,447,349	0.0655	570,257	0	-223,300	-223,300	>1.00
	1983	52,044,121	.040000	0.03158	0.00475	-438,211	46,471	0	-438,211	158,491,470	0.0544	621,535	0	-661,511	-661,511	0.94
	1984	52,634,574	.030000	0.04363	0.00841	717,409	763,881	0	717,409	211,126,044	0.0517	763,053	0	55,898	55,898	>1.00
AR	1981	37,208,159	.074268	0.06788	0.00647	-237,686	0	0	-237,686	37,208,159	0.0679	240,737	0	-237,686	-237,686	>1.00
	1982	24,586,499	.057134	0.07027	0.00800	322,968	322,968	0	322,968	61,794,658	0.0688	310,873	0	85,283	85,283	>1.00
	1983	24,866,313	.040000	0.04856	0.00721	212,856	535,824	0	212,856	86,660,971	0.0630	358,867	0	298,138	298,138	>1.00
	1984	28,755,157	.030000	0.03802	0.00693	230,616	766,440	0	230,616	115,416,128	0.0568	410,482	0	528,755	528,755	0.78
AZ	1981	18,204,168	.066681	0.08278	0.00973	293,069	293,069	1,696	291,373	18,204,168	0.0828	177,127	1,696	291,373	293,069	0.60
	1982	21,336,453	.053341	0.11603	0.01054	1,337,561	1,630,630	1,158,028	179,533	39,540,621	0.1007	286,265	1,159,724	470,906	1,630,630	0.18
	1983	39,230,909	.040000	0.10030	0.01251	2,365,624	3,996,254	1,901,898	463,725	78,771,530	0.1005	568,165	3,061,622	934,631	3,996,254	0.14
	1984	42,758,808	.030000	0.09656	0.01174	2,846,026	6,842,280	2,533,487	312,539	121,530,338	0.0991	758,158	5,595,110	1,247,170	6,842,280	0.11
CA	1981	1,270,296,772	.040000	0.06761	0.00843	35,072,894	35,072,894	17,457,244	17,615,650	1,270,296,772	0.0676	10,708,602	17,457,244	17,615,650	35,072,894	0.31
	1982	1,366,989,822	.040000	0.06001	0.00790	27,353,466	62,426,360	19,951,197	7,402,269	2,637,286,594	0.0637	15,208,461	37,408,441	25,017,919	62,426,360	0.24
	1983	1,493,164,856	.040000	0.04806	0.00560	12,034,909	74,461,269	8,502,920	3,531,989	4,130,451,450	0.0580	17,355,567	45,911,361	28,549,908	74,461,269	0.23
	1984	1,586,346,359	.030000	0.05177	0.00796	34,534,760	108,996,029	27,777,862	6,756,899	5,716,797,809	0.0563	21,463,104	73,689,223	35,306,807	108,996,029	0.20
CO	1981	47,081,958	.042135	0.08245	0.01024	1,898,109	1,898,109	1,105,023	793,086	47,081,958	0.0825	482,119	1,105,023	793,086	1,898,109	0.25
	1982	45,283,369	.041067	0.06603	0.00697	1,130,409	3,028,518	975,572	154,837	92,365,327	0.0744	576,245	2,080,595	947,923	3,028,518	0.19
	1983	51,766,123	.040000	0.06223	0.00673	1,150,761	4,179,279	990,985	159,775	144,131,450	0.0700	673,373	3,071,580	1,107,698	4,179,279	0.16
	1984	53,629,580	.030000	0.04618	0.00544	867,727	5,047,005	768,230	99,496	197,761,030	0.0636	733,857	3,839,811	1,207,195	5,047,005	0.15
CT	1981	102,601,922	.070950	0.07400	0.00401	312,936	312,936	0	312,936	102,601,922	0.0740	411,434	0	312,936	312,936	>1.00
	1982	105,097,773	.055475	0.06360	0.00902	853,919	1,166,855	0	853,919	207,699,695	0.0687	1,033,415	0	1,166,855	1,166,855	0.89
	1983	108,706,080	.040000	0.04401	0.00422	435,911	1,602,767	0	435,911	316,405,775	0.0602	1,130,659	0	1,602,767	1,602,767	0.71
	1984	111,939,465	.030000	0.03393	0.00456	439,922	2,042,689	1,998	437,924	428,345,240	0.0534	1,240,541	1,998	2,040,690	2,042,689	0.61
DC	1981	44,362,691	.162980	0.13564	0.00946	-1,212,876	0	0	-1,212,876	44,362,691	0.1356	419,671	0	-1,212,876	-1,212,876	0.35
	1982	43,215,977	.101490	0.17123	0.01282	3,013,882	3,013,882	657,676	2,356,206	87,578,668	0.1532	695,034	657,676	1,143,330	1,801,006	0.39
	1983	40,036,549	.040000	0.13150	0.01316	3,663,344	6,677,226	3,371,960	291,384	127,615,217	0.1464	872,167	4,029,636	1,434,715	5,464,350	0.16
	1984	37,300,887	.030000	0.11219	0.01038	3,065,760	9,742,986	2,930,739	135,021	164,916,104	0.1387	954,246	6,960,375	1,569,735	8,530,110	0.11
DE	1981	16,034,496	.120495	0.11276	0.01705	-124,027	0	0	-124,027	16,034,496	0.1128	273,388	0	-124,027	-124,027	>1.00
	1982	14,158,437	.080248	0.11875	0.02287	545,128	545,128	0	545,128	30,192,933	0.1156	423,780	0	421,101	421,101	>1.00
	1983	13,617,760	.040000	0.09371	0.01596	731,410	1,276,538	369,059	362,351	43,810,693	0.1088	476,263	369,059	783,452	1,152,511	0.41
	1984	13,785,238	.030000	0.07791	0.01637	660,451	1,936,989	576,954	83,496	57,595,931	0.1014	527,020	946,013	866,949	1,812,962	0.29
FL	1981	121,842,954	.050798	0.07925	0.00528	3,466,676	3,466,676	2,408,397	1,058,279	121,842,954	0.0793	643,331	2,408,397	1,058,279	3,466,676	0.19
	1982	119,632,382	.045399	0.06030	0.00543	1,782,642	5,249,318	1,336,974	445,668	241,475,336	0.0699	914,254	3,745,371	1,503,947	5,249,318	0.17

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Table 3-9. Application of Rule D to states (continued)

		Annual Statistics				Rule A Disallowance*		RULE D								
								Annual Values								
STATE	Year	Fed Contrib	Target	R-hat	s.e.	Annual (if positive)	Cumulated	Cash	Book	Fed Contrib	R-hat	sigma(D)	Cash	Book	Total	cv
	1983	138,762,474	.040000	0.04523	0.00353	725,728	5,975,046	523,472	202,255	380,237,810	0.0609	1,037,205	4,268,843	1,706,203	5,975,046	0.17
	1984	144,962,663	.030000	0.05354	0.00686	3,412,421	9,387,467	2,754,905	657,516	525,200,473	0.0588	1,436,911	7,023,748	2,363,719	9,387,467	0.15
GA	1981	105,505,310	.065285	0.06517	0.00497	-12,133	0	0	-12,133	105,505,310	0.0652	524,361	0	-12,133	-12,133	>1.00
	1982	113,975,476	.052642	0.05143	0.00746	-138,138	0	0	-138,138	219,480,786	0.0580	998,945	0	-150,271	-150,271	>1.00
	1983	125,472,468	.040000	0.05725	0.00488	2,164,400	2,164,400	86,732	2,077,668	344,953,254	0.0577	1,171,670	86,732	1,927,397	2,014,129	0.58
	1984	133,858,211	.030000	0.08177	0.00614	4,252,675	6,417,075	3,825,760	426,915	478,811,465	0.0589	1,431,193	3,912,492	2,354,312	6,266,804	0.23
HI	1981	46,619,415	.074989	0.10098	0.01210	1,211,685	1,211,685	283,749	927,936	46,619,415	0.1010	564,095	283,749	927,936	1,211,685	0.47
	1982	43,937,529	.057495	0.08217	0.01418	1,084,159	2,295,844	629,536	454,622	90,556,944	0.0919	840,461	913,285	1,382,558	2,295,844	0.37
	1983	43,207,080	.040000	0.06901	0.01258	1,253,437	3,549,281	989,503	263,935	133,764,024	0.0845	1,000,908	1,902,788	1,646,493	3,549,281	0.28
	1984	41,402,140	.030000	0.06653	0.01231	1,512,420	5,061,701	1,311,255	201,165	175,166,164	0.0802	1,123,196	3,214,044	1,847,658	5,061,701	0.22
IA	1981	83,979,881	.065241	0.04260	0.00440	-1,901,388	0	0	-1,901,388	83,979,881	0.0426	369,511	0	-1,901,388	-1,901,388	0.19
	1982	70,266,429	.052620	0.04492	0.00531	-541,052	0	0	-541,052	154,246,310	0.0437	525,122	0	-2,442,440	-2,442,440	0.21
	1983	80,125,076	.040000	0.03430	0.00550	-456,713	0	0	-456,713	234,371,386	0.0405	685,536	0	-2,899,153	-2,899,153	0.24
	1984	87,801,937	.030000	0.03662	0.00472	581,249	581,249	0	581,249	322,173,323	0.0394	801,066	0	-2,317,904	-2,317,904	0.35
ID	1981	14,481,785	.042925	0.09065	0.01878	691,143	691,143	243,756	447,387	14,481,785	0.0907	271,968	243,756	447,387	691,143	0.39
	1982	13,153,079	.041462	0.05430	0.01167	168,859	860,002	102,522	66,337	27,634,864	0.0733	312,294	346,278	513,724	860,002	0.36
	1983	14,024,312	.040000	0.02977	0.00975	-143,469	860,002	-190,554	47,085	41,659,176	0.0587	340,917	155,724	560,809	716,534	0.48
	1984	13,849,779	.030000	0.09495	0.01924	899,543	1,759,546	748,558	150,985	55,508,955	0.0677	432,702	904,282	711,794	1,616,077	0.27
IL	1981	390,914,682	.127286	0.08254	0.00703	-17,491,868	0	0	-17,491,868	390,914,682	0.0825	2,748,130	0	-17,491,868	-17,491,868	0.16
	1982	401,104,833	.082430	0.08243	0.00907	0	0	0	0	792,019,515	0.0825	4,559,322	0	-17,491,868	-17,491,868	0.26
	1983	411,830,534	.040000	0.06816	0.00782	11,597,148	11,597,148	0	11,597,148	1,203,850,049	0.0776	5,582,036	0	-5,894,721	-5,894,721	0.95
	1984	421,853,209	.030000	0.06497	0.00667	14,752,207	26,349,355	0	14,752,207	1,625,703,258	0.0743	6,251,110	0	8,857,486	8,857,486	0.71
IN	1981	83,266,989	.040000	0.04135	0.00525	112,410	112,410	0	112,410	83,266,989	0.0414	437,152	0	112,410	112,410	>1.00
	1982	78,402,824	.040000	0.03856	0.00486	-112,900	112,410	0	-112,900	161,669,813	0.0400	579,906	0	-490	-490	>1.00
	1983	82,652,384	.040000	0.04852	0.00474	704,198	816,609	0	704,198	244,322,197	0.0429	699,841	0	703,709	703,709	0.99
	1984	91,286,467	.030000	0.03963	0.00304	879,089	1,695,697	344,353	534,736	335,608,664	0.0420	752,854	344,353	1,238,445	1,582,797	0.48
KS	1981	47,251,492	.040903	0.08117	0.00937	1,902,676	1,902,676	1,174,358	728,318	47,251,492	0.0812	442,746	1,174,358	728,318	1,902,676	0.23
	1982	42,607,920	.040452	0.02813	0.00627	-525,015	1,902,676	-647,330	122,315	89,859,412	0.0560	517,102	527,028	850,633	1,377,661	0.38
	1983	47,801,194	.040000	0.05111	0.00825	531,071	2,433,747	311,930	219,141	137,660,606	0.0543	650,318	838,958	1,069,774	1,908,732	0.34
	1984	43,957,502	.030000	0.05489	0.00963	1,094,102	3,527,849	887,430	206,672	181,618,108	0.0545	775,955	1,726,389	1,276,446	3,002,835	0.26
KY	1981	99,638,877	.081195	0.04974	0.00431	-3,134,141	0	0	-3,134,141	99,638,877	0.0497	429,444	0	-3,134,141	-3,134,141	0.14
	1982	83,326,419	.060597	0.03576	0.00421	-2,069,578	0	0	-2,069,578	182,965,296	0.0434	554,514	0	-5,203,719	-5,203,719	0.11
	1983	86,117,601	.040000	0.03420	0.00396	-499,482	0	0	-499,482	269,082,897	0.0404	650,987	0	-5,703,201	-5,703,201	0.11
	1984	95,131,099	.030000	0.04146	0.00409	1,090,202	1,090,202	0	1,090,202	364,213,996	0.0407	758,401	0	-4,612,999	-4,612,999	0.16
LA	1981	89,792,909	.087025	0.06705	0.00600	-1,793,613	0	0	-1,793,613	89,792,909	0.0671	538,757	0	-1,793,613	-1,793,613	0.30
	1982	85,012,672	.063512	0.06163	0.00636	-159,994	0	0	-159,994	174,805,581	0.0644	763,279	0	-1,953,607	-1,953,607	0.39
	1983	80,125,076	.040000	0.05675	0.00699	1,342,095	1,342,095	0	1,342,095	254,930,657	0.0620	946,720	0	-611,512	-611,512	>1.00
	1984	93,291,207	.030000	0.05793	0.00597	2,605,344	3,947,439	186,973	2,418,371	348,221,864	0.0609	1,098,394	186,973	1,806,859	1,993,831	0.55

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Table 3-9. Application of Rule D to states (continued)

STATE	Year	Annual Statistics: Fed Contrib	Target	R-hat	s.e.	Rule A Disallowance*: Annual (if positive)	Cumulated	RULE D: Annual Values: Cash	Book	Fed Contrib	R-hat	sigma(D)	Cash	Book	Total	cv
MA	1981	266,657,338	.119126	0.09260	0.00839	-7,073,353	0	0	-7,073,353	266,657,338	0.0926	2,237,255	0	-7,073,353	-7,073,353	0.32
	1982	250,846,885	.079563	0.07362	0.00613	-1,490,783	0	0	-1,490,783	517,504,223	0.0834	2,714,738	0	-8,564,136	-8,564,136	0.32
	1983	223,000,695	.040000	0.11434	0.01028	16,577,872	16,577,872	2,168,748	14,409,124	740,504,918	0.0927	3,553,184	2,168,748	5,844,988	8,013,736	0.44
	1984	203,367,441	.030000	0.07757	0.00741	9,674,189	26,252,061	9,170,239	503,950	943,872,359	0.0895	3,859,537	11,338,987	6,348,938	17,687,925	0.22
MD	1981	113,146,541	.103615	0.11553	0.00741	1,325,512	1,325,512	0	1,325,512	113,146,541	0.1155	838,416	0	1,325,512	1,325,512	0.63
	1982	106,521,840	.071907	0.08218	0.00778	1,094,299	2,419,811	480,558	613,741	219,668,381	0.0994	1,178,877	480,558	1,939,253	2,419,811	0.49
	1983	112,256,768	.040000	0.05270	0.00424	1,425,661	3,845,472	1,273,565	152,096	331,925,149	0.0836	1,271,337	1,754,123	2,091,349	3,845,472	0.33
	1984	114,551,324	.030000	0.05665	0.00600	3,052,793	6,898,264	2,766,739	286,054	446,476,473	0.0767	1,445,230	4,520,861	2,377,403	6,898,264	0.21
ME	1981	40,429,640	.074661	0.07881	0.00930	167,743	167,743	0	167,743	40,429,640	0.0788	375,996	0	167,743	167,743	>1.00
	1982	41,341,291	.057330	0.04095	0.00591	-677,170	167,743	0	-677,170	81,770,931	0.0597	448,407	0	-509,428	-509,428	0.88
	1983	44,763,143	.040000	0.04548	0.00776	245,302	413,045	0	245,302	126,534,074	0.0546	567,211	0	-264,126	-264,126	>1.00
	1984	48,837,175	.030000	0.04144	0.00950	556,697	971,742	0	558,697	175,371,249	0.0510	732,790	0	294,572	294,572	>1.00
MI	1981	549,635,657	.074685	0.07284	0.00755	-1,014,078	0	0	-1,014,078	549,635,657	0.0728	4,149,749	0	-1,014,078	-1,014,078	>1.00
	1982	532,150,982	.057343	0.08235	0.00592	13,307,500	13,307,500	3,722,826	9,584,673	1,081,786,639	0.0775	5,210,088	3,722,826	8,570,595	12,293,422	0.42
	1983	566,088,345	.040000	0.09144	0.00546	29,119,584	42,427,084	27,729,659	1,389,926	1,647,874,984	0.0823	6,055,028	31,452,485	9,960,521	41,413,006	0.15
	1984	615,275,503	.030000	0.08011	0.00591	30,831,455	73,258,540	29,173,352	1,658,104	2,263,150,487	0.0817	7,062,994	60,625,836	11,618,625	72,244,462	0.10
MN	1981	134,920,297	.040000	0.04423	0.00795	570,713	570,713	0	570,713	134,920,297	0.0442	1,072,616	0	570,713	570,713	>1.00
	1982	127,746,141	.040000	0.03028	0.00709	-1,241,692	570,713	0	-1,241,692	262,666,436	0.0374	1,403,864	0	-670,980	-670,980	>1.00
	1983	140,175,501	.040000	0.02567	0.00389	-2,008,715	570,713	0	-2,008,715	402,841,939	0.0333	1,506,044	0	-2,679,695	-2,679,695	0.56
	1984	151,030,687	.030000	0.02014	0.00314	-1,489,163	570,713	0	-1,489,163	553,872,626	0.0297	1,578,945	0	-4,168,857	-4,168,857	0.38
MO	1981	116,840,365	.080665	0.07085	0.00674	-1,146,788	0	0	-1,146,788	116,840,365	0.0709	787,504	0	-1,146,788	-1,146,788	0.69
	1982	105,937,011	.060332	0.04772	0.00596	-1,336,078	0	0	-1,336,078	222,777,396	0.0599	1,009,361	0	-2,482,866	-2,482,866	0.41
	1983	113,032,839	.040000	0.03431	0.00382	-643,157	0	0	-643,157	335,810,235	0.0513	1,097,838	0	-3,126,023	-3,126,023	0.35
	1984	120,007,550	.030000	0.03709	0.00524	850,854	850,854	0	850,854	455,817,785	0.0475	1,265,183	0	-2,275,169	-2,275,169	0.56
MS	1981	48,171,208	.090413	0.06909	0.00689	-1,027,155	0	0	-1,027,155	48,171,208	0.0691	331,900	0	-1,027,155	-1,027,155	0.32
	1982	42,745,195	.065207	0.04738	0.00737	-762,019	0	0	-762,019	90,916,403	0.0589	457,805	0	-1,789,173	-1,789,173	0.26
	1983	43,781,804	.040000	0.03491	0.00744	-222,849	0	0	-222,849	134,698,207	0.0511	561,700	0	-2,012,023	-2,012,023	0.28
	1984	44,672,252	.030000	0.02027	0.00317	-434,661	0	0	-434,661	179,370,459	0.0434	579,276	0	-2,446,684	-2,446,684	0.24
MT	1981	12,019,670	.078326	0.04923	0.01078	-349,724	0	0	-349,724	12,019,670	0.0492	129,572	0	-349,724	-349,724	0.37
	1982	12,363,665	.059163	0.02547	0.00913	-416,569	0	0	-416,569	24,383,335	0.0372	171,845	0	-766,293	-766,293	0.22
	1983	15,494,757	.040000	0.02456	0.00914	-239,239	0	0	-239,239	39,878,092	0.0323	222,683	0	-1,005,532	-1,005,532	0.22
	1984	17,468,216	.030000	0.06910	0.01515	683,007	683,007	0	683,007	57,346,308	0.0435	345,867	0	-322,525	-322,525	>1.00
NC	1981	106,567,740	.066062	0.05420	0.00418	-1,264,107	0	0	-1,264,107	106,567,740	0.0542	445,453	0	-1,264,107	-1,264,107	0.35
	1982	96,970,814	.053031	0.03261	0.00337	-1,960,847	0	0	-1,960,847	203,538,554	0.0440	552,468	0	-3,224,953	-3,224,953	0.17
	1983	103,724,778	.040000	0.02684	0.00379	-1,365,018	0	0	-1,365,018	307,263,332	0.0382	678,058	0	-4,589,971	-4,589,971	0.15
	1984	102,984,223	.030000	0.03485	0.00437	499,473	499,473	0	499,473	410,247,555	0.0374	813,818	0	-4,090,498	-4,090,498	0.20
ND	1981	9,854,398	.040000	0.03089	0.00884	-89,774	0	0	-89,774	9,854,398	0.0309	87,113	0	-89,774	-89,774	0.97

Table 3-9. Application of Rule D to states (continued)

STATE	Year	Annual Statistics: Fed Contrib	Target	R-hat	s.e.	Rule A Disallowance*: Annual (if positive)	Cumulated	RULE D: Annual Values: Cash	Book	Fed Contrib	R-hat	sigma(D)	Cash	Book	Total	cv
	1982	8,921,303	.040000	0.01909	0.00547	-186,544	0	0	-186,544	18,775,701	0.0253	99,850	0	-276,318	-276,318	0.36
	1983	9,248,940	.040000	0.02071	0.00741	-178,412	0	0	-178,412	28,024,641	0.0238	121,108	0	-454,730	-454,730	0.27
	1984	9,718,794	.030000	0.04687	0.01375	163,956	163,956	0	163,956	37,743,435	0.0297	180,347	0	-290,774	-290,774	0.62
NE	1981	27,006,307	.044331	0.05470	0.01241	280,028	280,028	0	280,028	27,006,307	0.0547	335,148	0	280,028	280,028	>1.00
	1982	28,287,670	.042166	0.09594	0.01658	1,521,141	1,801,170	853,667	667,474	55,293,977	0.0758	575,990	853,667	947,503	1,801,170	0.32
	1983	31,391,923	.040000	0.04679	0.00904	213,151	2,014,321	104,394	108,757	86,685,900	0.0653	642,103	958,061	1,056,260	2,014,321	0.32
	1984	32,168,889	.030000	0.06896	0.01252	1,253,300	3,267,621	1,062,711	190,589	118,854,789	0.0663	757,963	2,020,771	1,246,849	3,267,621	0.23
NH	1981	16,882,118	.086656	0.06589	0.01241	-350,574	0	0	-350,574	16,882,118	0.0659	209,507	0	-350,574	-350,574	0.60
	1982	14,571,771	.063328	0.05855	0.01130	-69,624	0	0	-69,624	31,453,889	0.0625	266,470	0	-420,198	-420,198	0.63
	1983	14,073,658	.040000	0.04340	0.00722	47,850	47,850	0	47,850	45,527,547	0.0566	285,187	0	-372,348	-372,348	0.77
	1984	12,883,437	.030000	0.07522	0.01513	582,589	630,439	0	582,589	58,410,984	0.0607	345,438	0	210,241	210,241	>1.00
NJ	1981	270,515,844	.075481	0.08021	0.00764	1,279,269	1,279,269	0	1,279,269	270,515,844	0.0802	2,066,741	0	1,279,269	1,279,269	>1.00
	1982	256,603,933	.057740	0.07341	0.00747	4,020,984	5,300,253	663,319	3,357,665	527,119,777	0.0769	2,818,805	663,319	4,636,934	5,300,253	0.53
	1983	248,958,007	.040000	0.06364	0.00576	5,885,367	11,185,620	5,319,831	565,536	776,077,784	0.0726	3,162,596	5,983,150	5,202,470	11,185,620	0.28
	1984	245,446,764	.030000	0.05130	0.00640	5,228,016	16,413,636	4,621,607	606,409	1,021,524,548	0.0675	3,531,234	10,604,757	5,808,879	16,413,636	0.22
NM	1981	32,394,291	.045037	0.12386	0.01413	2,553,415	2,553,415	1,800,447	752,968	32,394,291	0.1239	457,731	1,800,447	752,968	2,553,415	0.18
	1982	30,773,114	.042519	0.10524	0.01095	1,930,120	4,483,536	1,748,092	182,029	63,167,405	0.1148	568,387	3,548,539	934,997	4,483,536	0.13
	1983	29,869,817	.040000	0.06025	0.01031	604,864	5,088,399	476,445	128,419	93,037,222	0.0973	646,453	4,024,984	1,063,416	5,088,399	0.13
	1984	34,686,013	.030000	0.05914	0.00787	1,010,750	6,099,150	919,827	90,924	127,723,235	0.0869	701,726	4,944,811	1,154,339	6,099,150	0.12
NV	1981	6,196,357	.040000	0.02260	0.00674	-107,817	0	0	-107,817	6,196,357	0.0226	41,763	0	-107,817	-107,817	0.39
	1982	6,023,800	.040000	0.01255	0.00590	-165,353	0	0	-165,353	12,220,157	0.0176	54,839	0	-273,170	-273,170	0.20
	1983	5,434,218	.040000	0.02691	0.00880	-71,134	0	0	-71,134	17,654,375	0.0205	72,761	0	-344,304	-344,304	0.21
	1984	5,084,327	.030000	0.02081	0.00973	-46,725	0	0	-46,725	22,738,702	0.0206	87,986	0	-391,029	-391,029	0.23
NY	1981	755,115,221	.071713	0.08002	0.00626	6,272,742	6,272,742	0	6,272,742	755,115,221	0.0800	4,727,021	0	6,272,742	6,272,742	0.75
	1982	835,083,462	.055856	0.07958	0.00708	19,811,520	26,084,262	13,632,025	6,179,495	1,590,198,683	0.0798	7,569,749	13,632,025	12,452,237	26,084,262	0.29
	1983	883,636,254	.040000	0.09381	0.00911	47,548,467	73,632,729	41,823,460	5,725,007	2,473,834,937	0.0848	11,049,996	55,455,486	18,177,243	73,632,729	0.15
	1984	957,340,305	.030000	0.07114	0.00794	39,384,980	113,017,709	35,499,474	3,885,506	3,431,175,242	0.0810	13,412,006	90,954,960	22,062,749	113,017,709	0.12
OH	1981	333,931,792	.076891	0.08866	0.00835	3,930,043	3,930,043	0	3,930,043	333,931,792	0.0887	2,788,330	0	3,930,043	3,930,043	0.71
	1982	334,115,763	.058446	0.07609	0.00821	5,895,139	9,825,182	3,390,870	2,504,269	668,047,555	0.0824	3,911,436	3,390,870	6,434,312	9,825,182	0.40
	1983	359,726,189	.040000	0.05609	0.00535	5,787,994	15,613,176	5,051,320	736,674	1,027,773,744	0.0732	4,359,262	8,442,190	7,170,986	15,613,176	0.28
	1984	401,826,269	.030000	0.06385	0.00475	13,601,819	29,214,995	12,944,575	657,245	1,429,600,013	0.0706	4,758,803	21,386,764	7,828,231	29,214,995	0.16
OK	1981	58,315,715	.040000	0.06587	0.01023	1,508,628	1,508,628	527,270	981,357	58,315,715	0.0659	596,570	527,270	981,357	1,508,628	0.40
	1982	44,318,866	.040000	0.03813	0.00684	-82,876	1,508,628	-195,777	112,901	102,634,581	0.0539	665,203	331,493	1,094,258	1,425,751	0.47
	1983	46,169,858	.040000	0.04051	0.00859	23,547	1,532,174	-156,179	179,725	148,804,439	0.0497	774,458	175,314	1,273,984	1,449,298	0.53
	1984	49,398,453	.030000	0.03021	0.00497	10,374	1,542,548	-52,108	62,482	198,202,892	0.0449	812,441	123,206	1,336,466	1,459,672	0.56
OR	1981	61,574,104	.098113	0.06772	0.01097	-1,871,422	0	0	-1,871,422	61,574,104	0.0677	675,468	0	-1,871,422	-1,871,422	0.36
	1982	52,881,730	.069056	0.07069	0.01088	86,409	86,409	0	86,409	114,455,834	0.0691	887,293	0	-1,785,013	-1,785,013	0.50
	1983	52,844,417	.040000	0.05983	0.00989	1,047,905	1,134,314	0	1,047,905	167,300,251	0.0662	1,029,773	0	-737,108	-737,108	>1.00

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Table 3-9. Application of Rule D to states (continued)

		Annual Statistics				Rule A Disallowance *		RULE D								
								Annual Values								
STATE	Year	Fed Contrib	Target	R-hat	s.e.	Annual (if positive)	Cumulated	Cash	Book	Fed Contrib	R-hat	sigma(D)	Cash	Book	Total	cv
	1984	57,654,583	.030000	0.04617	0.00724	932,275	2,066,588	0	932,275	224,954,834	0.0610	1,111,157	0	195,166	195,166	>1.00
PA	1981	421,504,157	.122190	0.09046	0.00562	-13,374,327	0	0	-13,374,327	421,504,157	0.0905	2,368,853	0	-13,374,327	-13,374,327	0.18
	1982	420,207,729	.081095	0.08537	0.00794	1,796,388	1,796,388	0	1,796,388	841,711,886	0.0879	4,091,865	0	-11,577,939	-11,577,939	0.35
	1983	416,640,265	.040000	0.09093	0.01005	21,219,489	23,015,877	10,739	21,208,749	1,258,352,151	0.0889	5,854,596	10,739	9,630,810	9,641,550	0.61
	1984	405,621,898	.030000	0.09062	0.00857	24,588,799	47,604,676	23,019,090	1,569,709	1,663,974,049	0.0893	6,808,827	23,029,830	11,200,520	34,230,349	0.20
RI	1981	43,270,544	.097739	0.06251	0.06251	-1,524,378	0	0	-1,524,378	43,270,544	0.0625	2,704,842	0	-1,524,378	-1,524,378	>1.00
	1982	40,285,720	.068870	0.05684	0.01039	-484,637	0	0	-484,637	83,556,264	0.0598	2,737,036	0	-2,009,015	-2,009,015	>1.00
	1983	39,028,872	.040000	0.06197	0.01139	857,464	857,464	0	857,464	122,585,136	0.0605	2,772,902	0	-1,151,551	-1,151,551	>1.00
	1984	41,316,420	.030000	0.03700	0.00679	289,215	1,146,679	0	289,215	163,901,556	0.0546	2,796,583	0	-862,336	-862,336	>1.00
SC	1981	56,158,502	.060519	0.07840	0.00593	1,004,170	1,004,170	456,352	547,818	56,158,502	0.0784	333,020	456,352	547,818	1,004,170	0.33
	1982	53,583,594	.050259	0.08892	0.00739	2,071,595	3,075,766	1,768,287	303,308	109,742,096	0.0835	517,402	2,224,640	851,126	3,075,766	0.17
	1983	53,676,391	.040000	0.07085	0.00801	1,655,917	4,731,682	1,400,409	255,508	163,418,487	0.0794	672,726	3,625,048	1,106,634	4,731,682	0.14
	1984	54,875,425	.030000	0.07784	0.00533	2,625,240	7,356,922	2,525,171	100,070	218,293,912	0.0790	733,558	6,150,219	1,206,704	7,356,922	0.10
SD	1981	11,866,284	.045230	0.04631	0.01448	12,816	12,816	0	12,816	11,866,284	0.0463	171,824	0	12,816	12,816	>1.00
	1982	11,399,541	.042615	0.03705	0.00764	-63,438	12,816	0	-63,438	23,265,825	0.0418	192,636	0	-50,623	-50,623	>1.00
	1983	11,982,862	.040000	0.02112	0.00498	-226,236	12,816	0	-226,236	35,248,687	0.0348	201,667	0	-276,859	-276,859	0.73
	1984	11,749,893	.030000	0.02909	0.00878	-10,692	12,816	0	-10,692	46,998,580	0.0333	226,522	0	-287,552	-287,552	0.79
TN	1981	59,079,920	.059800	0.08950	0.00680	1,754,674	1,754,674	1,093,806	660,868	59,079,920	0.0895	401,743	1,093,806	660,868	1,754,674	0.23
	1982	51,010,176	.049900	0.04912	0.00612	-39,788	1,754,674	-215,861	176,073	110,090,096	0.0708	508,779	877,945	836,941	1,714,886	0.30
	1983	55,639,492	.040000	0.04458	0.00442	254,829	2,009,502	162,184	92,645	165,729,588	0.0620	565,098	1,040,129	929,586	1,969,715	0.29
	1984	58,341,333	.030000	0.04281	0.00503	747,352	2,756,855	629,482	117,870	224,070,921	0.0570	636,751	1,669,611	1,047,456	2,717,067	0.23
TX	1981	87,575,396	.059120	0.07506	0.00779	1,395,952	1,395,952	273,713	1,122,239	87,575,396	0.0751	682,212	273,713	1,122,239	1,395,952	0.49
	1982	75,565,483	.049560	0.08364	0.00826	2,575,272	3,971,223	2,176,439	398,833	163,140,879	0.0790	924,664	2,450,152	1,521,072	3,971,223	0.23
	1983	94,659,889	.040000	0.06927	0.00923	2,770,695	6,741,918	2,199,075	571,620	257,800,768	0.0754	1,272,153	4,649,227	2,092,692	6,741,918	0.19
	1984	102,446,485	.030000	0.05686	0.00744	2,751,713	9,493,631	2,404,849	346,863	360,247,253	0.0702	1,483,012	7,054,076	2,439,555	9,493,631	0.16
UT	1981	34,319,580	.040000	0.04873	0.01181	299,610	299,610	0	299,610	34,319,580	0.0487	405,314	0	299,610	299,610	>1.00
	1982	32,754,359	.040000	0.04991	0.00879	324,596	624,206	0	324,596	67,073,939	0.0493	497,164	0	624,206	624,206	0.80
	1983	37,207,755	.040000	0.05651	0.01261	614,300	1,238,506	113,981	500,319	104,281,694	0.0519	683,602	113,981	1,124,525	1,238,506	0.55
	1984	35,943,708	.030000	0.05763	0.00928	993,125	2,231,630	866,398	126,727	140,225,402	0.0534	760,639	980,379	1,251,252	2,231,630	0.34
VA	1981	99,068,525	.091475	0.03589	0.00427	-5,506,724	0	0	-5,506,724	99,068,525	0.0359	423,023	0	-5,506,724	-5,506,724	0.08
	1982	93,924,089	.065737	0.04055	0.00477	-2,365,666	0	0	-2,365,666	192,992,614	0.0382	616,172	0	-7,872,390	-7,872,390	0.08
	1983	95,619,852	.040000	0.03757	0.00545	-232,356	0	0	-232,356	288,612,466	0.0380	806,996	0	-8,104,746	-8,104,746	0.10
	1984	93,253,012	.030000	0.03455	0.00459	424,301	424,301	0	424,301	381,865,478	0.0371	913,484	0	-7,680,445	-7,680,445	0.12
VT	1981	26,751,544	.043153	0.05157	0.01339	225,168	225,168	0	225,168	26,751,544	0.0518	358,203	0	225,168	225,168	>1.00
	1982	25,837,630	.041577	0.04520	0.00799	93,610	318,777	0	93,610	52,589,174	0.0484	413,435	0	318,777	318,777	>1.00
	1983	25,020,867	.040000	0.07881	0.01853	971,060	1,289,837	267,966	703,093	77,610,041	0.0582	621,198	267,966	1,021,871	1,289,837	0.48
	1984	27,659,366	.030000	0.05834	0.01127	783,866	2,073,704	662,424	121,442	105,269,407	0.0583	695,023	930,391	1,143,313	2,073,704	0.34

Table 3-9. Application of Rule D to states (continued)

		Annual Statistics				Rule A Disallowance*		RULE D								
								Annual Values								
STATE	Year	Fed Contrib	Target	R-hat	s.e.	Annual (if positive)	Cumulated	Cash	Book	Fed Contrib	R-hat	sigma(D)	Cash	Book	Total	cv
WA	1981	118,607,888	.058243	0.09333	0.01236	4,161,595	4,161,595	1,750,036	2,411,559	118,607,888	0.0933	1,465,993	1,750,036	2,411,559	4,161,595	0.35
	1982	119,737,415	.049122	0.06438	0.00685	1,826,953	5,988,548	1,475,174	351,780	238,345,303	0.0788	1,679,841	3,225,209	2,763,339	5,988,548	0.28
	1983	130,783,014	.040000	0.04775	0.00589	1,013,568	7,002,117	736,884	276,685	369,128,317	0.0678	1,848,039	3,962,093	3,040,024	7,002,117	0.26
	1984	147,030,923	.030000	0.04113	0.00526	1,636,454	8,638,571	1,380,985	255,469	516,159,240	0.0602	2,003,339	5,343,078	3,295,493	8,638,571	0.23
WI	1981	221,181,560	.087083	0.08238	0.00714	-1,040,217	0	0	-1,040,217	221,181,560	0.0824	1,579,236	0	-1,040,217	-1,040,217	>1.00
	1982	235,839,352	.063541	0.06479	0.00648	294,563	294,563	0	294,563	457,020,912	0.0733	2,197,613	0	-745,654	-745,654	>1.00
	1983	275,661,151	.040000	0.05078	0.00662	2,966,114	3,260,677	0	2,966,114	732,682,063	0.0648	2,856,515	0	2,220,460	2,220,460	>1.00
	1984	296,267,085	.030000	0.06602	0.00712	10,671,540	13,932,218	7,050,667	3,620,873	1,028,949,148	0.0652	3,550,963	7,050,667	5,841,334	12,892,001	0.28
WV	1981	41,068,616	.088803	0.07361	0.01314	-623,955	0	0	-623,955	41,068,616	0.0736	539,642	0	-623,955	-623,955	0.86
	1982	38,295,429	.064401	0.08245	0.00791	691,194	691,194	0	691,194	79,364,045	0.0779	618,847	0	67,239	67,239	>1.00
	1983	38,464,472	.040000	0.02980	0.00614	-392,338	691,194	0	-392,338	117,828,517	0.0622	662,381	0	-325,099	-325,099	>1.00
	1984	52,953,559	.030000	0.04808	0.00633	957,400	1,648,595	0	957,400	170,782,076	0.0578	742,365	0	632,301	632,301	>1.00
WY	1981	4,235,182	.040000	0.13747	0.01261	412,803	412,803	324,951	87,852	4,235,182	0.1375	53,406	324,951	87,852	412,803	0.13
	1982	4,317,706	.040000	0.04771	0.01253	33,290	446,093	-3,911	37,201	8,552,888	0.0922	76,020	321,040	125,053	446,093	0.17
	1983	5,590,715	.040000	0.07688	0.01584	206,186	652,278	139,250	66,936	14,143,603	0.0861	116,711	460,289	191,989	652,278	0.18
	1984	6,069,734	.030000	0.05559	0.01435	155,324	807,603	107,753	47,571	20,213,337	0.0770	145,629	568,042	239,560	807,603	0.18
Total	1981					72,162,950	72,162,950						28,900,797	-19,685,383	9,215,414	
	1982					94,967,566	167,130,516						82,560,681	6,305,223	88,865,904	
	1983					181,696,733	348,827,249						188,240,317	75,204,990	263,445,307	
	1984					230,099,299	578,926,548						368,130,578	123,432,787	491,563,365	

*Computed by simple application of Rule A. For states AZ and TX, these differ from the disallowances actually assessed (see Table 3–4), and for other states differ slightly from those shown in Table 3–4 because of variations in treatment of rounding errors.

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APPENDIX A

DESCRIPTION OF THE THREE TEST POPULATIONS AND THE SAMPLING PROCEDURE USED IN SIMULATIONS

The test populations consist of the cases included in the Federal subsamples for the year ending September 30, 1982, for three groups of states. The states used were:

Population A:	Illinois, New Jersey, Ohio, Pennsylvania
Population B:	Maryland, Michigan, South Carolina, Texas
Population C:	Arkansas, Colorado, Hawaii, Nebraska, Oregon, West Virginia

For each test population, the states chosen provide a sample of approximately 1500 cases that could be used as a test population from which samples could be drawn, with replacement, to study some of the characteristics of various sampling and estimation procedures for AFDC.

The following tables give some of the characteristics of each of the three test populations. Tables A-1 through A-3 provide summary measures. Tables A-1A through A-3C list the individual cases, by type.

From each population, simple random samples simulating state QC samples of various specified sizes were drawn in the following way. For each test population, the cases for which payment errors (ineligible, overpayment, or underpayment) were found by the state QC or by the Federal review were termed "error cases." Let P denote the proportion of error cases in the population, and let n denote the specified size of the state sample.

The number of error cases to be included in the state sample was determined by a random draw from the binomial distribution whose parameters are

P and n. That number of error cases was then drawn as a simple random sample, with replacement, from the set of error cases in the test population.

For the balance of the state sample, no error cases were involved. Consequently, the balance of the sample was drawn as a simple random sample of *payments* from the normal distribution whose mean and variance are those of the payments for the set of non-error cases of the population.

A Federal subsample of n' was drawn from each state sample. Let p_S denote the proportion of error cases in the state sample that was selected. The number of error cases to be included in the Federal subsample was determined by a random draw from the binomial distribution whose parameters are p_S and n'. That number of error cases in the state sample was then selected for the Federal subsample as a simple random sample, without replacement.

Subsamples of the non-error cases in the state sample did not have to be drawn, since estimates of the average overpayment per case, or of its variance, do not depend on the payment values of the non-error cases in the Federal subsample.

Except as otherwise specified, the statistics given in this report are based on repeated simple random samples from the test populations. Listings of the various results for each repetition of the sampling are available. Other sampling and estimation procedures can be applied if desired.

Table A-1. Statistics for Population A

Type of case	Number	Percent
Total cases	1,478	100.00
Cases in which both the Federal and state findings were that there was no payment error	1,266	85.66
Cases in which payment errors were found either by the state QC or the Federal review	212	14.34
Cases which the state found ineligible. Table A-1A lists these cases, showing the monthly payment and the Federal finding for each case. In this table, underpayments are shown as zero (as they are treated in the analyses).	62	4.19
Cases in which the state found no error or only underpayment error, and for which the Federal review found an overpayment. Table A-2A lists these cases, showing the monthly payment and the Federal finding.	49	3.32
Other cases in which the state found an overpayment error. Table A-3A lists these cases, showing the monthly payment, the state finding, and the Federal finding.	101	6.83

Statistic	State finding	Federal finding
Average monthly payment	296.22	--
Variance of monthly payment	64,892.93	--
Standard deviation of monthly payment	254.74	--
Coefficient of variation of payments	0.86	--
Average monthly overpayment	17.19	21.62
Variance of overpayments	3,762.48	4,970.75
Standard deviation of overpayments	61.34	70.50
Coefficient of variation of overpayments	3.57	3.26
Skewness (μ^3/σ^3)	n/a	3.80
Kurtosis (μ^4/σ^4)	n/a	17.70
Percent of cases with overpayments	11.03	12.65

Correlation of state and Federal findings of overpayment errors	.828
Regression coefficient for the regression of the Federal findings of overpayment to the state finding	.952
Overpayment error rate	.0730

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Table A-2. Statistics for Population B

Type of case	Number	Percent
Total cases	1,480	100.00
Cases in which both the Federal and state findings were that there was no payment error	1,260	85.14
Cases in which payment errors were found either by the state QC or the Federal review	220	14.86
Cases which the state found ineligible. Table A-1B lists these cases, showing the monthly payment and the Federal finding for each case. In this table underpayments are shown as zero (as they are treated in the analyses).	76	6.14
Cases in which the state found no error or only underpayment error, and for which the Federal review found an overpayment. Table A-2B lists these cases, showing the monthly payment and the Federal finding.	43	2.91
Other cases in which the state found an overpayment error. Table A-3B lists these cases, showing the monthly payment, the state finding, and the Federal finding.	101	6.82

Statistic	State finding	Federal finding
Average monthly payment	210.06	--
Variance of monthly payment	14,633.67	--
Standard deviation of monthly payment	120.97	--
Coefficient of variation of payments	0.58	--
Average monthly overpayment	15.04	16.69
Variance of overpayments	3,175.10	3,487.75
Standard deviation of overpayments	56.35	59.06
Coefficient of variation of overpayments	3.75	3.54
Skewness (μ^3/σ^3)	n/a	4.90
Kurtosis (μ^4/σ^4)	n/a	32.10
Percent of cases with overpayments	11.96	13.11

Correlation of state and Federal findings of overpayment errors	.940
Regression coefficient for the regression of the Federal findings of overpayment to the state finding	.985
Overpayment error rate	.0795

Table A-3. Statistics for Population C

Type of case	Number	Percent
Total cases	1,525	100.00
Cases in which both the Federal and state findings were that there was no payment error	1,317	86.36
Cases in which payment errors were found either by the state QC or the Federal review	208	13.64
Cases which the state found ineligible. Table A-1C lists these cases, showing the monthly payment and the Federal finding for each case. In this table underpayments are shown as zero (as they are treated in the analyses).	68	4.46
Cases in which the state found no error or only underpayment error, and for which the Federal review found an overpayment. Table A-2C lists these cases, showing the monthly payment and the Federal finding.	54	3.54
Other cases in which the state found an overpayment error. Table A-3C lists these cases, showing the monthly payment, the state finding, and the Federal finding.	86	5.64

Statistic	State finding	Federal finding
Average monthly payment	254.66	--
Variance of monthly payment	37,495.08	--
Standard deviation of monthly payment	193.64	--
Coefficient of variation of payments	0.76	--
Average monthly overpayment	13.66	16.87
Variance of overpayments	3,312.03	4,365.03
Standard deviation of overpayments	57.55	66.07
Coefficient of variation of overpayments	4.21	3.92
Skewness (μ^3/σ^3)	n/a	4.50
Kurtosis (μ^4/σ^4)	n/a	24.70
Percent of cases with overpayments	10.10	11.21

Correlation of state and Federal findings of overpayment errors	.809
Regression coefficient for the regression of the Federal findings of overpayment to the state finding	.928
Overpayment error rate	.0662

Table A-1A. Cases in Population A that state found ineligible, with Federal finding

Amount overpaid		Amount overpaid	
State	Federal	State	Federal
129	129	270	270
250	250	318	318
153	153	302	302
[illegible]	368	302	302
30[illegible]	368	250	250
250	250	125	[illegible]5
250	250	434	434
302	302	319	0
348	348	273	273
273	273	273	273
360	360	273	273
137	137	360	360
273	273	263	263
360	360	216	216
360	360	216	216
360	360	216	216
273	273	216	0
360	360	216	216
350	350	216	216
273	273	111	111
216	216	263	263
216	216	131	131
216	216	395	395
216	216	321	321
111	111	273	273
216	216	321	321
263	263	172	172
216	216	265	265
262	262	387	387
318	318	172	172
381	381	360	360

Total cases	62
Cases with Federal zero	2

Table A-1B. Cases in Population B that state found ineligible, with Federal finding

Amount overpaid		Amount overpaid	
State	Federal	State	Federal
118	118	240	240
55	55	606	606
118	118	259	259
141	141	225	225
112	112	84	0
12	12	409	409
141	141	395	395
164	164	273	273
23	23	434	434
141	141	413	413
85	85	206	206
153	153	491	491
141	141	327	327
118	118	102	102
164	164	133	133
102	102	172	172
102	102	163	163
102	102	97	97
102	102	204	204
48	48	141	141
133	133	118	118
163	163	118	118
102	102	14	14
163	163	85	85
133	133	23	23
72	72	118	118
102	102	23	23
211	211	85	85
211	211	230	230
270	270	295	295
247	247	67	67
326	326	355	355
326	326	270	270
134	134	211	211
211	211	211	211
211	211	247	247
211	211	326	326
295	295	326	326

Number of cases	76
Cases with Federal zero	1

Table A-1C. Cases in Population C that state found ineligible, with Federal finding

Amount overpaid		Amount overpaid	
State	Federal	State	Federal
140	140	98	98
122	122	116	116
122	122	186	186
89	89	140	140
247	247	59	59
247	247	86	86
283	283	415	415
247	247	247	247
63	0	222	222
50	50	224	224
168	168	390	390
185	185	365	365
523	523	420	420
175	175	420	420
375	375	45	45
468	468	560	560
72	72	240	240
155	155	560	560
86	86	231	231
286	286	409	409
547	547	58	58
480	480	206	206
286	0	206	206
286	286	206	206
134	134	206	206
164	164	249	249
54	54	164	164
86	86	122	122
164	164	179	179
164	164	10	10
164	164	142	142
164	164	122	122
164	164	100	100
164	164	140	140

Total cases	68
Cases with Federal zero	2

Table A-2A. Cases in Population A for which the state found no error or only underpayment

Payment	Federal		Payment	Federal	
	Ineligible	Overpayment		Ineligible	Overpayment
302	302		221		0
240		12	236		0
236	236		250	250	
360		87	302	302	
195		68	357		0
360		132	236		0
414	414		334		165
234		0	477	477	
174		0	413	413	
324	324		324		100
216		105	263		245
263	263		216	216	
90		0	131		0
327		189	263		47
216		101	327		64
216	216		327	327	
224		0	263	263	
216		0	48		0
175	175		438		57
113		0	194		0
381		63	404		153
381	381		337		211
438		57	214		140
265		0	223		0
321		0			

Total cases	49
Federal finding:	
No overpayment cases	16
Ineligible cases	15
Other overpayment cases	18

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Table A-2B. Cases in Population B for which the state found no error or only underpayment

	Federal			Federal	
Payment	Ineligible	Overpayment	Payment	Ineligible	Overpayment
118		11	314		0
118		50	395		35
141		23	450		0
23		0	249		0
107		0	318		0
133	133		306		0
133	133		223		0
102	102		182		0
72	72		314		0
44		0	383		0
193		31	204		32
113		0	236		44
94		0	133	133	
326		284	106		0
270	270		118		10
422	422		118	118	
225		0	118		0
502		0	118	118	
29		0	131	131	
205		0	326		28
305		0	270	270	
386		56			

Total cases	43
Federal finding:	
No overpayment cases	21
Ineligible cases	11
Other overpayment cases	11

Table A-2C. Cases in Population C for which the state found no error or only underpayment

Payment	Federal Ineligible	Federal Overpayment	Payment	Federal Ineligible	Federal Overpayment
83		48	59		0
247		0	116	116	
130		0	264		0
76		0	62		0
434		0	856	856	
375	375		56		0
297		0	448	448	
57		0	210	210	
280		10	350	350	
140		0	286	286	
190		79	436		39
150		0	257		200
355	355		286		177
323		0	239		140
286		0	177	117	
150	150		134	134	
286	286		176		0
253		0	164		17
204		0	136		82
361		278	176		30
286	286		134		0
339		199	122		33
547		67	100		0
69		0	51	51	
98		0	20		0
65		0	100	100	
161		0	173		0

Total cases	54
Federal finding:	
No overpayment cases	26
Ineligible cases	14
Other overpayment cases	14

Table A-3A. Cases in Population A for which the state found eligible but overpayment

Payment	State overpayment	Federal Ineligible	Federal Overpayment	Payment	State overpayment	Federal Ineligible	Federal Overpayment
250	98		98	326	23		287
302	52		52	478	40		40
250	170		170	381	43		63
250	170		170	536	3	536	
302	62		62	395	51		51
225	192		192	264	89		89
225	72		72	714	200		200
80	9		9	368	8		58
649	424		424	309			52
153	73		73	250			24
302	52		52	250			170
237	65		65	242	40		40
250	30		30	368	66		66
502	60		60	302	222		222
236	56		56	700	51		51
468	54		54	302	52		52
360	87		87	284	80		80
246	136		0	378	54		54
360	87		0	414	54		54
188	166		166	414	54		54
414	54		54	522	54		54
522	54		54	360	90		90
273	136		136	311	65		15
273	136		136	414	54		0
273	136		136	246	136		136
273	136		136	180	41		41
360	87		0	263	47		0
360	91		91	216	99	216	
414	141		141	127	63		63
414	141		141	263	37		37
263	47		47	206	131		131
262	64		64	200	64		64
164	14		14	216	105		105
263	51		51	263	47		47
475	148		148	327	64		64
1105	104		104	167	18		18
263	152		152	341	84		84
263	47		47	424	43		43
327	64		64	384	63		63
381	301		301	481	43		43
302	47		47	335	73		73
536	98		98	253	12		0
286	55		55	385	63		63
438	120		120	438	194		194
451	144		119	94	43		43
381	63		63	327	73		73
318	129		129	74	34		34
441	13		13	262	90		90
436	57		57	224	220		220
234	46		0	84	22		22
318	86		44				

101 cases, of which 7 showed no Federal overpayment

Table A-3B. Cases in Population B for which the state found eligible but overpayment

Payment	State overpayment	Federal Ineligible	Federal Overpayment	Payment	State overpayment	Federal Ineligible	Federal Overpayment
139	63		0	318	11		11
121	43	121		568	76		76
118	47		47	354	106		106
164	14		37	106	87		87
110	12		12	327	68		68
183	53		62	568	76		76
102	28		28	418	76		76
184	21		21	406	59		59
163	129		129	506	18		56
193	127		127	253	9		9
270	177		112	421	13		11
685	42		42	276	23		0
211	79		73	241	52	241	
270	111		117	451	51		51
270	59		70	372	31		31
211	91		91	190	51		51
270	50		41	439	112		112
326	266		266	305	21		0
270	141		141	297	33		33
270	59		59	607	74		74
211	91		91	543	238		171
222	56		60	102	30		30
553	31		31	223	30		30
404	20		20	102	17		17
306	105		105	163	17		17
640	17		17	72	18		18
348	206		206	133	32		32
421	73		73	218	14		14
601	316		316	82	34		23
360	75		75	164	120		120
206	116		116	141	46		46
511	13		13	164	16		16
487	73		0	118	70		70
405	162		162	118	63		63
548	74		48	118	63		63
395	67		68	164	31		31
530	97		97	118	30		30
478	50		50	164	108		108
511	83		83	81	23		23
203	83		83	141	23		23
576	19		19	69	32		32
460	320		320	164	62		62
620	595		595	85	32		32
641	208		208	510	56		56
305	75		75	131	5		9
403	32		32	295	252		252
296	67		67	295	65		65
274	85		85	230	90		90
458	28		28	270	59		70
327	193		193	326	266		266
292	67		67				

101 cases, of which 4 showed no Federal overpayment

Table A-3C. Cases in Population C for which the state found eligible but overpayment

Payment	State overpayment	Federal Ineligible	Federal Overpayment	Payment	State overpayment	Federal Ineligible	Federal Overpayment
122	105		105	59	49		49
450	137		137	140	39		0
308	152		152	253	63		63
247	227		227	253	83		83
183	94		94	247	62		62
247	158		158	379	61		61
91	6		6	379	55		0
383	78		78	379	105		105
247	67		67	543	47		47
214	17		17	298	59		66
189	28		28	359	84		84
313	66		66	468	120		120
247	6		6	531	63		63
546	15		15	474	19		19
546	396		396	336	112		112
546	15		15	222	81		81
521	468		0	373	53		53
128	39		39	350	106		106
254	17		17	390	93		93
546	78		78	410	78		44
334	77		77	122	63		63
420	70		70	448	25		25
490	210		210	118	22		0
420	80		0	350	70	350	
350	70		70	174	10		0
164	18		0	286	200		200
203	9		0	339	48		48
301	8		8	403	33		33
323	15		15	376	30		30
763	55		55	266	18		18
286	200		200	222	19		19
329	53		53	212	52		52
281	75		75	134	116		116
134	44		44	134	44		44
164	43		43	98	66		66
164	18		18	206	30		30
90	64		64	90	17	90	
215	39		39	206	42		42
164	30		30	76	10		10
206	42		42	142	32		32
206	148		148	100	49		49
206	148		148	100	17		17
164	25		0	72	10		10

86 cases, of which 9 showed no Federal overpayment

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APPENDIX B

EVALUATION OF THE REGRESSION AND DIFFERENCE ESTIMATORS

Classical regression analysis assumes a linear relationship between the dependent and the independent variables, and that the dependent variable is (at least approximately) normally distributed for each value of the independent variable. As noted earlier in this report (Section 2.2), the requirements of classical regression analysis are reasonably well satisfied in the application of the regression estimator when one considers the fact that the "independent" variable is the Federal subsample mean of the error per case as determined by the state review and the "dependent" variable is the mean error per case as determined by the Federal re-review for the cases in the same subsample. Relationships between these means were illustrated in Section 2.2 (Figure 2-1) by scatter diagrams for 1000 samples drawn from Test Population A for each of four sample sizes. We include here similar scatter diagrams for the other two test populations which we have examined (Figures B-1 and B-2).

We emphasize that the linearity is not required for the regression estimator to be consistent (i.e., unbiased in large enough samples). However, the close approximation to linearity that is illustrated in the figures leads to negligible bias even for the smallest sizes of Federal subsamples. A little algebra brings out how the bias decreases with sample sizes, and becomes negligible for large enough samples.

The regression estimator of the mean error per case is

$$\bar{x}'' = \bar{x}' + b'\left(\bar{y} - \bar{y}'\right) .$$

Then, conditional on the state sample S, the expected value of $\bar{x}''$ is

$$E\left(\bar{x}'' \mid S\right) = \bar{x} + E\left\{b'\left(\bar{y} - \bar{y}'\right) \mid S\right\}$$

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Figure B-1. Mean findings of dollar error per case in 1000 independent samples for each of four sample sizes, Population B

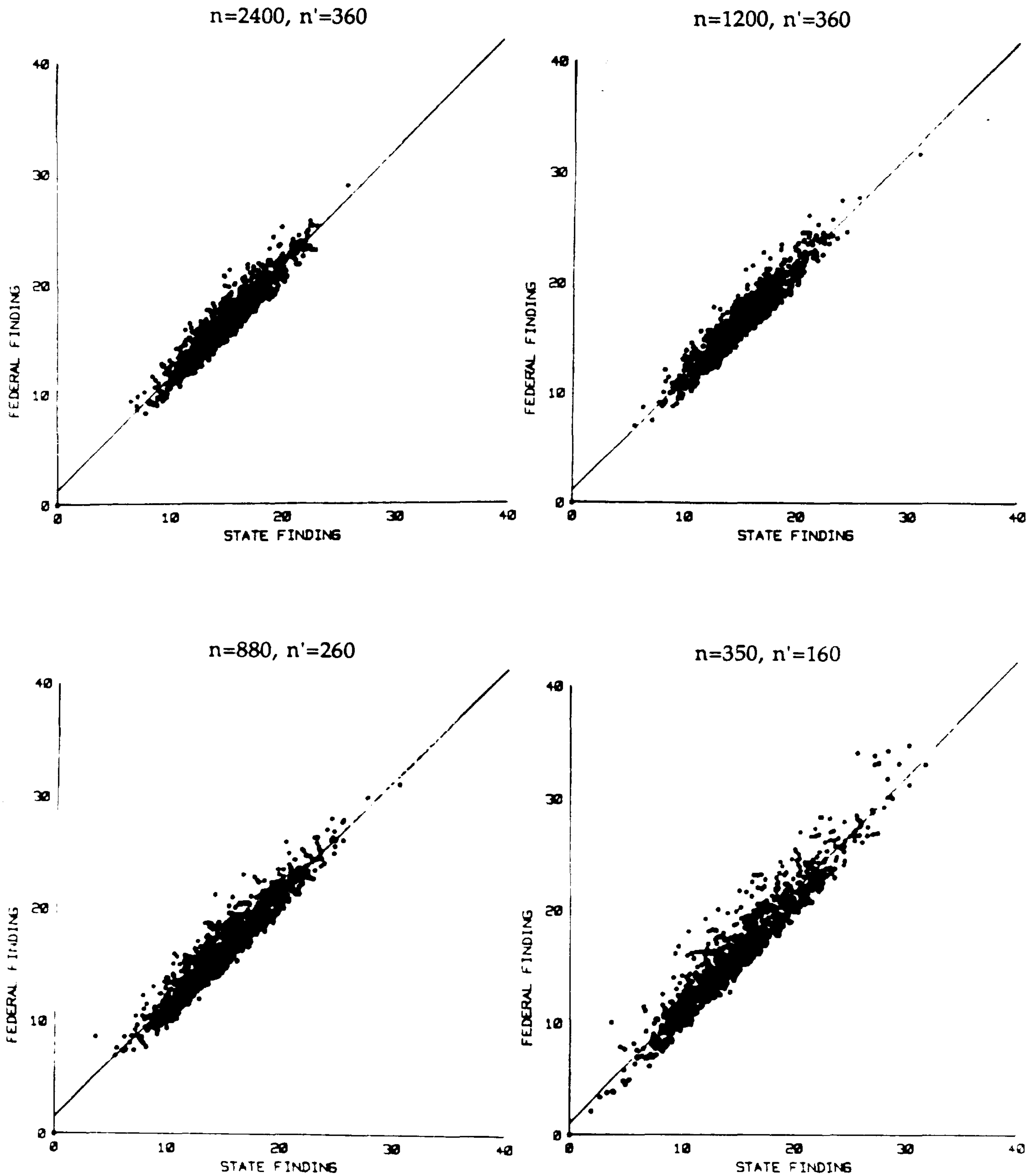
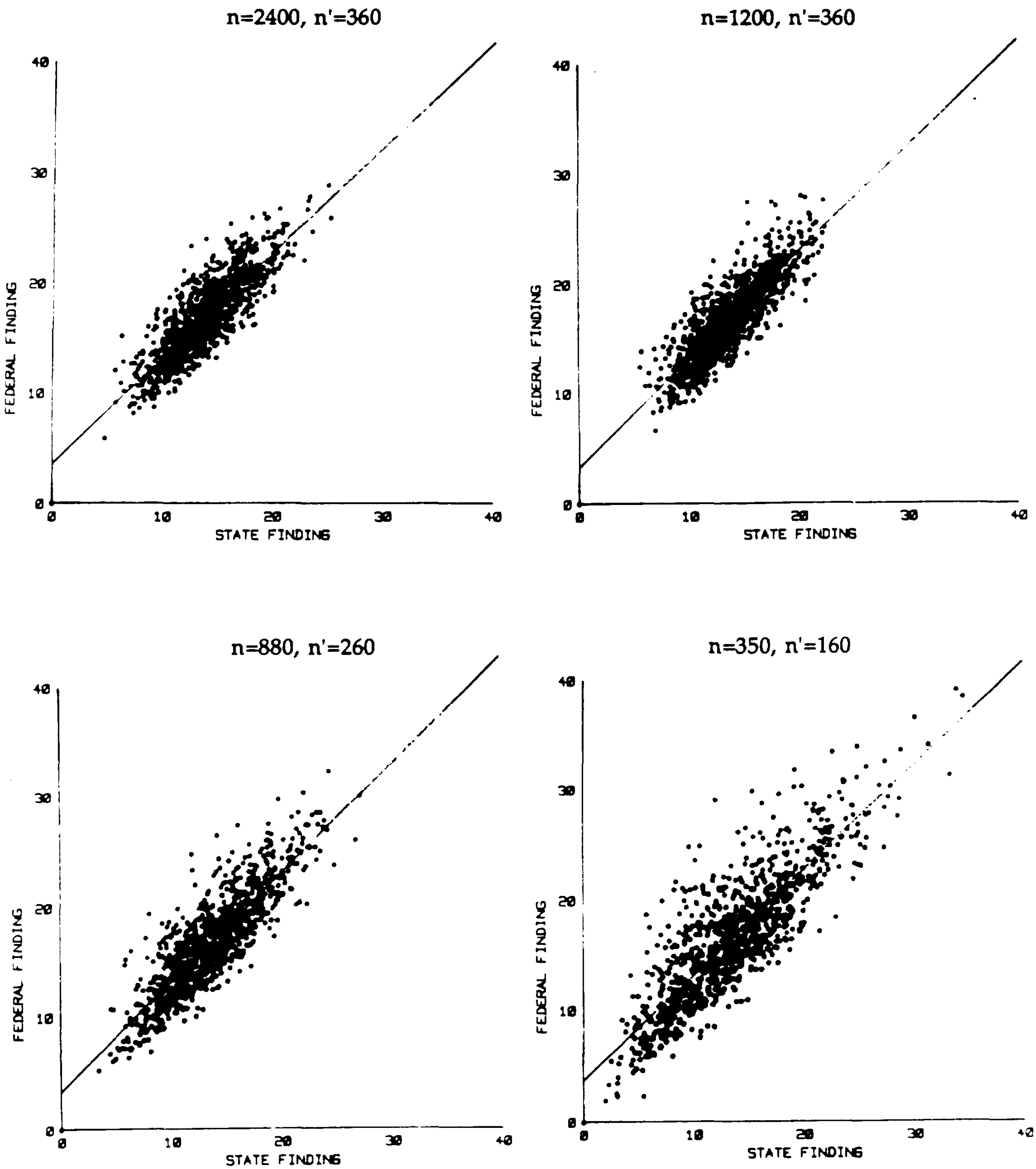


Figure B-2. Mean findings of dollar error per case in 1000 independent samples for each of four sample sizes, Population C



and therefore over all possible state samples,

$$E(\bar{x}'') = \bar{X} + E\,E\left\{b'(\bar{y} - \bar{y}') \mid s\right\}.$$

Thus, the bias of $\bar{x}''$ as an estimate of $\bar{X}$ is

$$E\,E\left\{b'(\bar{y} - \bar{y}') \mid s\right\}.$$

We note that

$$E\left\{b'(\bar{y} - \bar{y}') \mid s\right\} = -\,\mathrm{Cov}\,(b', \bar{y}' \mid s)$$

$$= \rho_{b'\bar{y}'|s}\,\sigma_{b'|s}\,\sigma_{\bar{y}'|s}\ .$$

Since each of these standard deviations is of order $1/\sqrt{n'}$ and the correlation coefficient is no greater than 1 in absolute value, the bias is of order no greater than $1/n'$. Thus, the bias decreases with increases in the size of the Federal subsample and is negligible for sufficiently large samples.

Also, since the bias of $\bar{x}''$ (and of $\hat{R}$) is of the order $\frac{1}{n'}$, and the standard error is of the order $\frac{1}{\sqrt{n'}}$, the ratio of the bias to the standard error decreases with increasing sample size and is negligible for large enough samples.

We have also examined the distribution of the residuals, $d_i = \bar{x}'_i - (a + b\,\bar{y}'_i)$, for the lines of regression shown in Figure 2-1 in Chapter 2, and in Figures B-1 and B-2 above. The coefficients a and b of the regression line are computed from the known population parameters. Summary measures for the distributions of the 1000 residuals are given in Table B-4 for each of the four sample sizes for the three test populations. The summary measures in the table are defined as follows:

Mean $\quad \bar{d} = \Sigma\, d_i/1000$

Standard deviation $\sigma = \left[\Sigma (d_i - \bar{d})^2/1000\right]^{1/2}$

Skewness $\dfrac{\Sigma (d_i - \bar{d})^3}{1000} / \sigma^3$

Kurtosis $\dfrac{\Sigma (d_i - \bar{d})^4}{1000} / \sigma^4$

It is seen from the measures of skewness and kurtosis that the distributions show some moderate departure from normal, but are reasonably close to the values for a normal distribution of 0 for skewness and 3 for kurtosis.

B.1 Comparison of the Regression and Difference Estimators

We initially had some concern that the approximations that are involved in the regression estimator and the estimator of its variance may not be totally satisfactory because of the relatively small sizes of the Federal subsamples. The so-called difference estimator, on the other hand, provides unbiased estimates for any sample size and an unbiased estimate of its variance is available. We have, therefore, on occasion, considered the use of the difference estimator to replace the regression estimator. To compare these alternative estimators in the context of the AFDC quality control program, we have simulated sampling from Population A, described in Appendix A.

The regression estimator $\hat{R}$ is defined by

$$\hat{R} = \{\bar{x}' + b(\bar{y} - \bar{y}')\} / \bar{t}$$

and the difference estimator $\tilde{R}$ is defined by

$$\tilde{R} = \{\bar{x}' + k(\bar{y} - \bar{y}')\} / \bar{t}$$

where

$\bar{x}' = \Sigma x_i/n'$	is the average overpayment in the Federal subsample whose size is n', the average being computed over all cases in the subsample, regardless of whether there was an overpayment, as determined by the Federal review;
$\bar{y} = \Sigma y_i/n$	is the average overpayment in the state QC sample whose size is n, as determined by the state review;
$\bar{y}' = \Sigma y_i/n'$	is the average overpayment in the Federal sample, as determined by the state QC review;

$$b = \Sigma(x_i - \bar{x}')(y_i - \bar{y}')/\Sigma(y_i - \bar{y}')^2$$

	is an estimate of the regression coefficient, as estimated from the Federal subsample;
k	is a constant which, if it were equal to the true value of the regression coefficient, would minimize the variance of the difference estimator;
x_i, y_i	denote respectively the Federal and state determination of the overpayment for case i;
$\bar{t}$	is the average AFDC payment per case in the state QC sample.

From Population A, 1000 samples were drawn using simple random sampling (see Appendix A) for various sample sizes to simulate state QC samples, and from each sample a simple random subsample was drawn to simulate a Federal subsample. For each sample, the regression estimate and three difference estimates using three values of the constant k were computed, as well as the appropriate estimates of their variances. The standard error of the regression estimate $\hat{R}$ is estimated by

$$s_{\hat{R}} = s_x\{(1 - r^2(1-n'/n))/n'\}^{1/2} / \bar{t}$$

and the standard error of the difference estimate $\tilde{R}$ for a given value of k is estimated by

$$s_{\tilde{R}(k)} = \{(1-n'/n)(s_x^2 + k^2 s_y^2 - 2krs_x s_y)/n' + s^2/n\}^{1/2},$$

where

$$s_x^2 = \Sigma(x_i - \bar{x}')^2/(n'-1)$$

is the unit variance of overpayments as determined by the Federal review for the cases in the Federal subsample, and

$$s_y^2 = \Sigma(y_i - \bar{y}')^2/(n'-1)$$

is the unit variance of overpayments as determined by the state QC review for the cases in the Federal subsample.

Results of the simulation comparing the estimators are shown in Tables B-1 and B-2.

The true value of the overpayment error rate in Population A is .0730. Table B-1 shows that the average value of $\hat{R}$, estimated from the 1000 independent samples is very close to the true value for each of the three sample sizes. This indicates, as discussed more fully below, that the bias, if any, of the regression estimator is trivial for this population, even for the small sample sizes considered. The fact that the average values of the difference estimates $\tilde{R}$ differ slightly from the true value is due to sampling variation, for the difference estimator can be shown to be unbiased.

Table B-2 shows, for each of the four estimators and for each of the three sample sizes, the variance (i.e, the square of the standard error) of the estimated payment error rate, the average of the estimated variances given by the formulas above, and the standard deviation of the estimated variances. We note that the variances, estimated by 1000 repetitions of the sampling procedure, differ very little among the four estimators, for each of the sample sizes. The average of

the variance estimates also appears to differ little among the four estimators of the payment error rate. The fact that the average of the variance estimates is slightly smaller than the estimate of the true variance is attributable to sampling variation, since the variance estimator for the difference estimator of the payment error rate can be shown to be unbiased. For each size of sample, the four estimates of the payment error rate and of its variance were made from the same sample and hence are expected to be similar. The reasonable interpretation of these results is that the bias of the estimator of the variance of the regression estimate is trivial.

We note also that the standard deviation of the estimated variance increases with a decrease in the sample size, approximately as predicted by statistical theory.

B.2 Validity of the Regression Estimator

Examination of Table B-3 indicates that while the average value of the estimated payment error rate is very close to the population value, in 11 of the 12 independent estimates the average value is somewhat less than the true payment error rate for the population. The largest of the individual differences is 2.3 times its estimated standard error. These results suggest a small downward bias of the regression estimator. However, the indicated biases are all so small that they contribute trivially (less than 1 percent) to the mean square error, and are so small that they can be neglected. There is no such indication of a bias in the estimates of the standard error of the estimated payment error rate.

We emphasize that the absence of appreciable bias in the regression estimator or in the estimator of its variance does not suffice to ensure that the estimator of the payment error rate is satisfactory. The *variability* of the estimated variance is quite large, as can be seen from the simulation results presented in Table B-3. Hence, much of the variation of the standard error between years for a given state, and much of the variation between states in a given year, may be due simply to sampling error.

Various sample sizes have been used in this appendix and elsewhere in this report. One set of sample sizes, in particular,

$$n = 1200 \qquad n' = 180$$
$$n = 500 \qquad n' = 80$$
$$n = 300 \qquad n' = 50$$

was used in initial analyses. The largest of these sample sizes was intended to approximate the six-month sample sizes in use in the larger states. The smaller sample sizes were chosen to evaluate results with small Federal sample sizes even smaller than those in use. Later, in order to approximate more nearly many of the annual sample sizes currently in use in AFDC, additional sample sizes were used in the simulations, as follows:

$$n = 2400 \qquad n' = 360$$
$$n = 1200 \qquad n' = 360$$
$$n = 880 \qquad n' = 260$$
$$n = 350 \qquad n' = 160$$

These sample sizes were generally used in the more recent analyses.

Similarly, Population A was the only test population that was defined initially. Many of the earlier simulations used only that test population. Later, Test Populations B and C were defined, in order to examine the stability of the conclusions for various populations. Generally, the conclusions were found to be very similar for the test populations, and consequently, some of the analyses were limited to one or two test populations.

However, many of the simulations and analyses were carried through for all three test populations. For example, Tables C-2A through C-2C in Appendix C show a number of comparable simulation results for all three test populations. From those tables, we summarize in Table B-3 the regression estimates of the overpayment error rate for each of four sample sizes for each of the three test populations, and their estimated standard errors, and comparisons can be made with the true overpayment error rates that are being estimated.

Table B-1. Average values of the estimated payment error rate $\hat{R}$ and its estimated standard deviation based on 1000 independent samples from Population A, [illegible] estimator and sample size

Estimator	Average $\hat{R}$	Standard deviation	Average $\hat{R}$	Standard deviation	Average $\hat{R}$	Standard deviation
Regression	0.0727	0.0118	0.0727	0.0176	0.07[illegible]3	0.0228
Difference						
k=1	0.0[illegible]28	0.0117	0.0728	0.0173	0.0725	0.0222
k=.9	0.0728	0.0118	0.0727	0.0173	0.0726	0.0223
k=.8	0.0728	0.0120	0.0726	0.0176	0.0727	0.0228

Table B-2. Variance of the estimated payment error rate and the average of estimates, by estimator and sample size (based on 1000 independent samples from Population A)

	Sample size n=1200, n'=180			Sample size n=500, n'=80			Sample size n=300, n'=50		
Estimator	Variance	Average variance estimate	Standard deviation of variance	Variance	Average variance estimate	Standard deviation of variance	Variance	Average variance estimate	Standard deviation of variance
Regression	1.39E-04	1.30E-04	.6300E-04	3.10E-04	2.90E-04	2.06E-04	5.20E-04	4.70E-04	4.26E-04
Difference									
k=1	1.37E-04	1.31E-04	.6400E-04	2.99E-04	2.94E-04	2.08E-04	4.93E-04	4.79E-04	4.33E-04
k=.9	1.39E-04	1.31E-04	.6300E-04	2.99E-04	2.94E-04	2.07E-04	4.97E-04	4.79E-04	4.30E-04
k=.8	1.44E-04	1.35E-04	.6300E-04	3.10E-04	3.03E-04	2.07E-04	5.20E-04	4.94E-04	4.30E-04
Average	1.40E-04	1.32E-04	.6300E-04	3.05E-04	2.95E-04	2.07E-04	5.08E-04	4.81E-04	4.30E-04

Table B-3. Some summary statistics from 1000 simulations for Populations A, B, and C

	Sample size		Test population		
	n	n'	A	B	C
R			.07297	.07945	.06623
	2400	360			
$\bar{\hat{R}}$			.07306	.07893	.06592
$\hat{\sigma}_{\bar{\hat{R}}}$			.00025	.00023	.00028
$\hat{\sigma}_{\hat{R}}$			.00792	.00736	.00872
$\bar{s}_{\hat{R}}$			.00791	.00713	.00861
s.e. $(\bar{s}_{\hat{R}})$			.00004	.00004	.00007
s.e. $(s_{\hat{R}})$			.00138	.00139	.00227
	1200	360			
$\bar{\hat{R}}$			.07245	.07906	.06601
$\hat{\sigma}_{\bar{\hat{R}}}$			.00027	.00026	.00030
$\hat{\sigma}_{\hat{R}}$			.00839	.00807	.00937
$\bar{s}_{\hat{R}}$			.00884	.00895	.00966
s.e. $(\bar{s}_{\hat{R}})$			.00004	.00004	.00007
s.e. $(s_{\hat{R}})$			.00126	.00139	.00214
	880	260			
$\bar{\hat{R}}$			.07271	.07882	.06564
$\hat{\sigma}_{\bar{\hat{R}}}$			.00033	.00031	.00035
$\hat{\sigma}_{\hat{R}}$			.01036	.00973	.01091
$\bar{s}_{\hat{R}}$			.01033	.01040	.01116
s.e. $(\bar{s}_{\hat{R}})$			.00006	.00006	.00009
s.e. $(s_{\hat{R}})$			.00182	.00190	.00289
	350	160			
$\bar{\hat{R}}$			.07290	.07930	.06607
$\hat{\sigma}_{\bar{\hat{R}}}$			.00048	.00049	.00051
$\hat{\sigma}_{\hat{R}}$			.01513	.01560	.01624
$\bar{s}_{\hat{R}}$			.01451	.01544	.01552
s.e. $(\bar{s}_{\hat{R}})$			.00009	.00011	.00015
s.e. $(s_{\hat{R}})$			.00292	.00363	.00471

Definitions:

R — True payment error rate

$\hat{R}$ — Estimated error rate for a single sample

$\bar{\hat{R}}$ — Mean value of 1000 estimates of R

$\hat{\sigma}_{\hat{R}}$ — Estimated standard error of $\hat{R}$

$\hat{\sigma}_{\bar{\hat{R}}}$ — Estimated standard error of $\bar{\hat{R}}$

$s_{\hat{R}}$ — Estimated standard error of $\hat{R}$ for a single sample

$\bar{s}_{\hat{R}}$ — Mean estimate of the standard error of $\hat{R}$

s.e. $(s_{\hat{R}})$ — Estimated standard error of $s_{\hat{R}}$

s.e. $(\bar{s}_{\hat{R}})$ — Estimated standard error of $\bar{s}_{\hat{R}}$

Table B-4. Summary measures for distribution of residuals, for regression of $\overline{x}'$ on $\overline{y}'$

	Sample size			
	2400/360	1200/360	880/260	350/160
Population A				
Mean	0.000	0.000	0.000	0.000
Standard deviation	2.044	2.043	2.491	3.052
Skewness	0.383	0.353	0.485	0.538
Kurtosis	3.398	3.084	3.045	3.432
Population B				
Mean	0.000	0.000	0.000	0.000
Standard deviation	1.029	1.008	1.173	1.532
Skewness	0.776	0.823	0.885	1.122
Kurtosis	3.681	3.872	3.900	4.444
Population C				
Mean	0.000	0.000	0.000	0.000
Standard deviation	1.988	1.970	2.281	3.061
Skewness	0.480	0.572	0.636	0.845
Kurtosis	3.090	3.631	3.648	4.092

APPENDIX C

COMPUTATION OF CONFIDENCE INTERVALS

Confidence intervals for the payment error rate are produced in the current AFDC quality control program in the following way. An estimate of the standard error of the estimated payment error rate is computed by the formula given for $s_{\hat{R}}$ in Section 1.1 of Chapter 1 (Equation (3)) and also in Appendix B. The lower and upper bounds of the nominal confidence interval at a given confidence level are defined by $\hat{R} \pm t\, s_{\hat{R}}$, where, for example, t=1.96 for the 95 percent confidence level and t=1.645 for the 90 percent confidence level. These values of the coefficient t are appropriate if $\hat{R}$ were a mean estimated from a simple random sample from a normal distribution, and $s_{\hat{R}}$ its estimated standard error. This is a commonly used procedure. Such confidence intervals are referred to as nominal confidence intervals for the specified level of confidence (say 95 percent) because the actual probabilities may not conform to the specified level of confidence.

Suppose that the samples were large enough that $\hat{R}$ and $s_{\hat{R}}$ were approximately normally distributed and also large enough that the coefficient of variation of $s_{\hat{R}}$ was small (say less than .02). For a nominal confidence level of 95 percent, these conditions are sufficient for the actual probability to be close to 2.5 percent that the lower bound of the interval is greater than the value being estimated, 2.5 percent that the upper bound is less than the value being estimated, and 95 percent that the value being estimated is between the bounds. Similar statements hold for the 90 percent confidence interval. (See the attached Technical Note for Appendix C.)

For the QC samples in use in AFDC, the distribution of $\hat{R}$ appears to be reasonably close to normal, although still slightly skewed to the right and somewhat more skewed for the smaller sample sizes (see Figure 2-2 in Section 2.3 of the report). The distribution of $s_{\hat{R}}$ is also skewed but still reasonably approaching

normality (see Figure C-1). Moreover, and particularly relevant, is that the coefficient of variation of $s_{\hat{R}}$ is quite large, being several times larger than it would be if the estimate $\hat{R}$ were the sample mean of a normally distributed variable based on a sample of size n', and $s_{\hat{R}}$ were the associated estimate of its standard error. Also, $\hat{R}$ and $s_{\hat{R}}$ are positively correlated. The results are not sensitive to that correlation (which remains constant with increasing sample size), but are highly sensitive to the coefficient of variation of $s_{\hat{R}}$ (which decreases with increasing sample size).

Estimated values of the coefficient of variation $V_{s_{\hat{R}}}$ and of the correlation $\hat{\rho}$ of $\hat{R}$ and $s_{\hat{R}}$ for the regression estimator, for various sample sizes, drawn from Test Populations A, B, and C, are given in Table C-1.

Table C-1. Correlation of $\hat{R}$ and $s_{\hat{R}}$, coefficients of variation of $s_{\hat{R}}$ and of $\tilde{\beta}$, estimated from 1000 independent samples of Test Populations A, B, and C, for various sample sizes

Sample sizes			Population A			Population B			Population C		
n	n'	n'/n	$\hat{\rho}$	$V_{s_{\hat{R}}}$	$\tilde{\beta}$	$\hat{\rho}$	$V_{s_{\hat{R}}}$	$\tilde{\beta}$	$\hat{\rho}$	$V_{s_{\hat{R}}}$	$\tilde{\beta}$
2400	360	.15	.75	.18	48	.66	.20	59	.68	.27	106
1200	360	.30	.75	.14	29	.62	.16	38	.66	.22	71
880	260	.30	.76	.18	35	.61	.18	35	.68	.26	71
350	160	.46	.79	.20	27	.67	.24	38	.71	.30	59
1200	180	.15	.77	.25	46	.64	.27	54	NA	NA	NA
500	80	.16	.76	.37	45	.67	.39	50	NA	NA	NA
300	50	.17	.78	.48	47	.60	.50	51	NA	NA	NA

NA - not available.

These are estimated from 1000 independent samples for each population and for each sample size. As expected, for a given population, and with some sampling variability, the correlations are essentially constant over the various sample sizes, whereas the coefficients of variation of $s_{\hat{R}}$ decrease approximately as the square root of the Federal subsample size n' increases.

Note 1: Table C-1 also shows for each illustrative test population and sample size some values labeled $\tilde{\beta}$. These values provide another indicator of how much larger the variance of the variance estimates are than would be expected in estimating a mean from a simple random sample drawn from a normal population. Thus, for a simple random sample of n' drawn with replacement (from any distribution of a variable X), the relvariance of the sample estimate of the variance of the mean is approximately[1]

$$V^2_{s^2_{\bar{x}}} = \frac{\sigma^2_{s^2_{\bar{x}}}}{(\sigma^2)^2} = \frac{\beta - 1}{n'}$$

where σ^2 is the variance of the distribution,

$$s^2_{\bar{x}} = \sum (x_i - \bar{x})^2 / (n' - 1)\, n'$$

is the estimated variance of the sample mean, $\bar{x}$, for a simple random sample of n' (drawn from any distribution), and

$$\beta = \Sigma(x_i - 2)^4 / n\sigma^4.$$

For a normal distribution, β has the value 3, but may have considerably larger (or smaller) values for various non-normal distributions. Also, in general, the relvariance of $s_{\bar{x}}$ is approximately one-fourth of the relvariance of $s^2_{\bar{x}}$. If we substitute $\tilde{\beta}$ for β and $4\sigma^2_{s_{\bar{x}}} = \sigma^2_{s^2_{\bar{x}}}$ in the above equation we obtain

[1]Hansen, M.H., Hurwitz, W.N., and Madow, W.G. (1953), ***Sample Survey Methods and Theory***, Vol. I, Chapter 10, (New York: John Wiley & Sons). Theory for samples drawn with replacement provides a simple approximation for samples drawn without replacement provided the sampling fraction is small.

$$\tilde{\beta} = 4n'V_s^2 + 1 .$$

We have found it convenient, in Appendix E, to use these values of $\tilde{\beta}$ in obtaining rough approximations to the variance of state estimates of $s_{\hat{R}}^2$.

For AFDC-QC, a consequence of the large coefficient of variation of $s_{\hat{R}}$ and of the positive correlation of $\hat{R}$ and $s_{\hat{R}}$ is that the probability of the left tail (i.e., the probability that the lower confidence bound is above the value being estimated) is considerably less than the nominal probability; the probability of the right tail is considerably greater than the nominal probability. The technical note attached to this Appendix shows the expected frequency below, above, and covered by 95 percent and 90 percent nominal confidence intervals for the case in which both $\hat{R}$ and $s_{\hat{R}}$ are normally distributed and are positively correlated, for various values of the coefficient of variation of $s_{\hat{R}}$ and of the correlation of the two variables.

Figures C-2A to C-2D are scatter diagrams showing the relationship between the values of $\hat{R}$ and $s_{\hat{R}}$ for the 1000 samples drawn at each of four sample sizes for Population A. That the correlation between the variables is positive is clear. It is also quite clear that the joint distribution is reasonably close to normal. The ellipses in the diagram are such as to enclose a specified proportion of the points if the joint distribution were exactly normal. The inner ellipse would include 50 percent, the next would include 90 percent, the third would include 95 percent, and the outer ellipse would include 99 percent of the points. For the 1000 actual samples, the results were as follows:

Contour	Sample size			
	2400/360	1200/360	800/260	350/160
.50	491	506	495	508
.90	901	902	904	898
.95	957	950	951	950
.99	990	993	992	983

Thus, the observed frequencies approximate, reasonably closely, the proportions that are expected for the bivariate normal distribution. However, the moderate skewness of the marginal distribution of $\hat{R}$ and $s_{\hat{R}}$ is evident; in each case, there are more points in the right hand tail of the marginal distribution than in the left hand tail.

Tables C-2A, C-2B, and C-2C, which are based on 1000 independent samples drawn for Populations A, B, and C, respectively, show summary statistics of the current AFDC sample design. They also show some summary measures for specific confidence bounds and for the coverage of nominal confidence intervals based on the same 1000 samples.

The panel headed "CONFIDENCE BOUNDS" gives, for example, the value of $\hat{R}$ such that 2.5 percent of the estimates $\hat{R}$ fall below it. This value was estimated from the 1000 independent samples drawn from the specified population, using the state and Federal sample sizes specified in the column headings of the table. The 5 percent, 95 percent, and 97.5 percent points were similarly estimated from the same samples.

The next panel, headed "NOMINAL CONFIDENCE BOUNDS," gives the estimated means and variances of the bounds, the bounds being computed by the current AFDC procedure. The line labeled "Coverage" gives the estimated probability that the specified tail covers the true value, R. For example, for Population A with the sample size 2400/360, the probability that the nominal 2.5 percent point is greater than R is estimated to be 1.1 percent rather than the nominal 2.5 percent. Similarly, the probability that the nominal 97.5 percent point is less than R is estimated to be 5.3 percent rather than the nominal 2.5 percent. Consequently, the coverage of the corresponding 95 percent confidence interval is estimated to be 93.6 percent (i.e., 100 – 1.1 – 5.3) rather than the nominal 95 percent.

The panel of the tables that is headed "NOMINAL CONFIDENCE BOUNDS, MINIMUM rho" gives the results of a procedure we have considered (see Chapter 3 of this report and Appendix D) to reduce the effect of unusually low values of the estimated correlation, $\hat{\rho}$, between the state and Federal findings for the

same case. This may happen because of sampling variation. It could also happen if a state, inadvertently or not, does a poor job of evaluation in its QC operation. The procedure consists of replacing the estimated correlation by a constant value whenever the estimated correlation is less than that constant value. The constant value used in these computations was .8. The tables show that this has only a minor or negligible effect on the coverage properties of the resulting confidence intervals.

Table C-3 summarizes the coverage of the nominal 95 percent and 90 percent confidence intervals for the three populations and various sample sizes.

These results are reasonably close to expectations for samples large enough that both $\hat{R}$ and $s_{\hat{R}}$ are normally distributed, as shown in the Technical Note. They also conform to the general statement made above about the effect of the coefficient of variation of $s_{\hat{R}}$ and the correlation of $\hat{R}$ and $s_{\hat{R}}$. As seen from Table C-3, the coverage of the 95 percent and 90 percent confidence intervals is generally somewhat less than the nominal confidence coefficient, but reasonably close, especially for the larger sample sizes. They may reasonably be regarded as providing acceptable approximations to the nominal probabilities of 95 percent and 90 percent, and therefore can serve as useful measures of the precision of $\hat{R}$ as an estimate of R.

We note from Table C-3 that, for the variance estimator that imposes a minimum value of ρ, the coverage probabilities are essentially the same as for the variance estimator that uses the estimated ρ, although slightly farther from the nominal probabilities.

One way of circumventing or reducing the effect of the skewness of the distribution of $\hat{R}$ is to compute confidence intervals on a transformation of $\hat{R}$ whose distribution is more nearly symmetrical. If a transformation of $\hat{R}$, say $u=f(\hat{R})$, is normally distributed, and if an unbiased or consistent estimate of the standard error of u is available, one might have confidence bounds for the expected value of u whose probabilities are more nearly the nominal confidence levels. Those bounds could then be transformed by the inverse transformation, say g(u), to yield

confidence bounds for R with probabilities corresponding to the nominal confidence levels. We therefore simulated sampling from the test populations using the natural logarithm transformation $f(\hat{R})=\ln \hat{R}$.

The procedure used was the following: a sample simulating a simple random state sample of specified size n was drawn (with replacement) from the test population. A subsample simulating a Federal subsample of size n' was then drawn (without replacement) from the state sample. Each element of the state sample was assigned at random to exactly one of 90 "replicate sets." The 90 replicate sets were subdivided at random into 45 pairs of sets, each giving rise to a Jackknife replicate estimate of R. The Jackknife replicate estimate corresponding to a given pair is an estimate that uses the data in the state and Federal samples, but replaces a random one of the replicate sets in the given pair by the other replicate set of the same pair.

Let $\hat{R}_{(j)}$ denote the estimated payment error rate based on the i–th Jackknife replicate, for i=1,2,...,45. The Jackknife estimate of the variance of R is given by

$$s^2_{\hat{R}} = \Sigma_j \, (\hat{R}_{(j)} - \hat{R})^2 .$$

The estimate based on the full sample and each Jackknife replicate estimate was then subjected to the logarithmic transformation:

$$R^* = \ln \hat{R}$$

$$R^*_{(j)} = \ln \hat{R}_{(j)} .$$

The Jackknife estimator of the variance of R^* is then

$$s^2_{R^*} = \Sigma_j \, (R^*_{(j)} - R^*)^2 .$$

The confidence interval for the mathematical expectation of R^* at a specified confidence level is computed as $R^* \pm ks_{R^*}$, where the multiplier k is appropriate to the confidence level for a normal distribution. Denote

$$L_1^* = R^* - ks_{R^*}$$

$$L_2^* = R^* + ks_{R^*}$$

and let

$$L_1 = \exp(L_1^*)$$

$$L_2 = \exp(L_2^*).$$

Then L_1 and L_2 are taken to be the lower and upper bounds, respectively, of the confidence interval for the payment error rate, R.

For each of the four sample sizes the procedure was repeated 400 times. Table C-4 shows the estimated coverage probabilities of the intervals corresponding to the nominal 2.5 percent point, 5 percent point, 95 percent point, and 97.5 percent point, as well as the estimated coverage probability corresponding to the nominal 90 percent confidence interval. It also shows, for comparison, the coverage of confidence intervals computed by the conventional procedure described at the beginning of this Appendix.

Later, in order to obtain additional information on the validity of the logarithmic transformation, the procedure was repeated an additional 1500 times for Population A using the sample sizes n=2400, n'=360, and an additional 2000 times using the sample sizes n=350, n'=160. The combined results of the two sets of simulations are summarized in Table 2-6 of Section 2.4 of the report.

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Table C-2A. Population A: Summary statistics

STATISTIC R=.07297		2400/360	1200/360	880/260	350/160
Mean R'		0.073057	0.072446	0.0727088	0.072901
Variance of R'		6.279E-05	7.044E-05	1.073E-04	2.290E-04
Mean estimated variance of R'		6.446E-05	7.989E-05	1.100E-04	2.193E-04
Variance of estimated variance of R'		4.937E-10	5.456E-10	1.539E-09	7.606E-09
Mean estimated standard error of R'		0.007908	0.008845	0.010327	0.014513
Variance of estimated standard error of R'		1.916E-06	1.600E-06	3.311E-06	8.542E-06
CONFIDENCE BOUNDS					
2.5% point		0.058890	0.055498	0.052032	0.044432
5.0% point		0.060931	0.058098	0.055404	0.048894
95.0% point		0.085944	0.087887	0.090412	0.098589
97.5% point		0.088519	0.090339	0.094691	0.105241
NOMINAL CONFIDENCE BOUNDS					
2.5% point	Mean	0.057557	0.055110	0.052467	0.044455
	Variance	3.804E-05	4.623E-05	6.487E-05	1.263E-04
	Coverage	0.011	0.006	0.010	0.013
5.0% point	Mean	0.060048	0.057896	0.055720	0.049027
	Variance	4.102E-05	4.929E-05	6.997E-05	1.383E-04
	Coverage	0.024	0.028	0.028	0.031
95.0% point	Mean	0.086067	0.087000	0.089697	0.096775
	Variance	9.493E-05	1.002E-04	1.625E-04	3.659E-04
	Coverage	0.084	0.097	0.100	0.102
97.5% point	Mean	0.088558	0.089782	0.092951	0.101347
	Variance	1.023E-04	1.069E-04	1.751E-04	3.973E-04
	Coverage	0.053	0.059	0.066	0.075
NOMINAL CONFIDENCE BOUNDS, MINIMUM rho					
2.5% point	Mean	0.057933	0.055389	0.052892	0.044950
	Variance	4.037E-05	4.744E-05	6.767E-05	1.281E-04
	Coverage	0.013	0.008	0.014	0.016
5.0% point	Mean	0.060364	0.058130	0.056077	0.049442
	Variance	4.322E-05	5.043E-05	7.265E-05	1.403E-04
	Coverage	0.030	0.030	0.032	0.034
95.0% point	Mean	0.085751	0.086762	0.089341	0.096360
	Variance	9.018E-05	9.774E-05	1.563E-04	3.595E-04
	Coverage	0.084	0.098	0.100	0.107
97.5% point	Mean	0.088182	0.089503	0.092526	0.100852
	Variance	9.632E-05	1.038E-04	1.673E-04	3.893E-04
	Coverage	0.053	0.060	0.067	0.075

Note: Based on 1000 trials, for the regression estimate.

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Table C-2B. Population B: Summary statistics

STATISTIC R=.0794491		2400/360	1200/360	880/260	350/160
Mean R'		0.078925	0.079055	0.078815	0.079299
Variance of R'		5.413E-05	6.519E-05	9.470E-05	2.434E-04
Mean estimated variance of R'		5.278E-05	8.216E-05	1.119E-04	2.518E-04
Variance of estimated variance of R'		4.356E-10	6.840E-10	1.710E-09	1.351E-08
Mean estimated standard error of R'		0.007130	0.008953	0.010402	0.015442
Variance of estimated standard error of R'		1.945E-06	1.944E-06	3.619E-06	1.321E-05
CONFIDENCE BOUNDS					
2.5% point		0.064904	0.063426	0.060174	0.050866
5.0% point		0.066957	0.065943	0.062379	0.054757
95.0% point		0.090980	0.094013	0.097331	0.108100
97.5% point		0.094786	0.098049	0.100251	0.114120
NOMINAL CONFIDENCE BOUNDS					
2.5% point	Mean	0.064949	0.061507	0.058428	0.049033
	Variance	3.506E-05	4.662E-05	6.630E-05	1.482E-04
	Coverage	0.011	0.012	0.008	0.017
5.0% point	Mean	0.067195	0.064327	0.061705	0.053897
	Variance	3.711E-05	4.860E-05	6.899E-05	1.566E-04
	Coverage	0.032	0.030	0.033	0.036
95.0% point	Mean	0.090653	0.093783	0.095926	0.104702
	Variance	8.167E-05	9.230E-05	1.400E-04	4.016E-04
	Coverage	0.093	0.072	0.093	0.096
97.5% point	Mean	0.092899	0.096604	0.099203	0.109566
	Variance	8.815E-05	9.869E-05	1.509E-04	4.400E-04
	Coverage	0.067	0.042	0.055	0.062
NOMINAL CONFIDENCE BOUNDS, MINIMUM rho					
2.5% point	Mean	0.064957	0.061515	0.058461	0.049109
	Variance	3.504E-05	4.666E-05	6.654E-05	1.486E-04
	Coverage	0.011	0.012	0.009	0.017
5.0% point	Mean	0.067202	0.064334	0.061733	0.053961
	Variance	3.711E-05	4.863E-05	6.922E-05	1.570E-04
	Coverage	0.032	0.030	0.033	0.036
95.0% point	Mean	0.090647	0.093776	0.095898	0.104638
	Variance	8.162E-05	9.221E-05	1.395E-04	4.005E-04
	Coverage	0.093	0.072	0.093	0.096
97.5% point	Mean	0.092891	0.096595	0.099170	0.109490
	Variance	8.808E-05	9.858E-05	1.503E-04	4.386E-04
	Coverage	0.067	0.042	0.055	0.062

Note: Based on 1000 trials, for the regression estimate.

Table C-2C. Population C: Summary statistics

STATISTIC		2400/360	1200/360	880/260	350/160
R=.066230					
Mean R'		0.065917	0.066014	0.065643	0.066066
Variance of R'		7.605E-05	8.779E-05	1.191E-04	2.637E-04
Mean estimated variance of R'		7.964E-05	9.788E-05	1.331E-04	2.631E-04
Variance of estimated variance of R'		1.946E-09	2.005E-09	5.184E-09	2.738E-08
Mean estimated standard error of R'		0.008616	0.009656	0.011163	0.015517
Variance of estimated standard error of R'		5.147E-06	4.561E-06	8.373E-06	2.221E-05
CONFIDENCE BOUNDS					
2.5% point		0.049868	0.047857	0.044524	0.037619
5.0% point		0.052507	0.050076	0.047661	0.041242
95.0% point		0.081174	0.082178	0.085395	0.094651
97.5% point		0.084358	0.085426	0.088494	0.101151
NOMINAL CONFIDENCE BOUNDS					
2.5% point	Mean	0.049031	0.047088	0.043764	0.035653
	Variance	4.300E-05	5.446E-05	6.838E-05	1.373E-04
	Coverage	0.003	0.011	0.009	0.007
5.0% point	Mean	0.051745	0.050129	0.047281	0.040541
	Variance	4.550E-05	5.745E-05	7.218E-05	1.462E-04
	Coverage	0.014	0.021	0.020	0.028
95.0% point	Mean	0.080090	0.081898	0.084005	0.091591
	Variance	1.359E-04	1.428E-04	2.113E-04	5.015E-04
	Coverage	0.093	0.103	0.113	0.120
97.5% point	Mean	0.082804	0.084940	0.087522	0.096479
	Variance	1.507E-04	1.562E-04	2.341E-04	5.608E-04
	Coverage	0.060	0.080	0.084	0.087
NOMINAL CONFIDENCE BOUNDS, MINIMUM rho					
2.5% point	Mean	0.049899	0.047749	0.044648	0.036712
	Variance	4.481E-05	5.574E-05	6.906E-05	1.357E-04
	Coverage	0.010	0.011	0.011	0.009
5.0% point	Mean	0.052473	0.050685	0.048022	0.041429
	Variance	4.810E-05	5.924E-05	7.407E-05	1.472E-04
	Coverage	0.029	0.028	0.026	0.030
95.0% point	Mean	0.079361	0.081343	0.083264	0.090703
	Variance	1.220E-04	1.336E-04	1.957E-04	4.748E-04
	Coverage	0.098	0.104	0.116	0.124
97.5% point	Mean	0.081936	0.084278	0.086638	0.095420
	Variance	1.329E-04	1.443E-04	2.140E-04	5.260E-04
	Coverage	0.062	0.084	0.088	0.090

Note: Based on 1000 trials, for the regression estimate.

Table C-3. Estimated coverage of 95 percent and 90 percent nominal confidence intervals for three test populations, based on alternative regression estimators using the estimated ρ and a minimum ρ of .8

State n	Federal n'	Population A Estimated ρ	Population A Minimum ρ	Population B Estimated ρ	Population B Minimum ρ	Population C Estimated ρ	Population C Minimum ρ
95 percent nominal confidence interval							
2400	360	0.936	0.934	0.922	0.922	0.937	0.928
1200	360	0.935	0.932	0.946	0.946	0.909	0.905
800	260	0.924	0.919	0.937	0.936	0.907	0.901
350	160	0.912	0.909	0.921	0.921	0.906	0.901
90 percent nominal confidence interval							
2400	360	0.892	0.886	0.875	0.875	0.893	0.873
1200	360	0.875	0.872	0.898	0.898	0.876	0.868
800	260	0.872	0.868	0.874	0.874	0.867	0.858
350	160	0.867	0.859	0.868	0.868	0.852	0.846

Note: Based on 1000 independent replicate samples from each population for each sample size. The same replicate was used with the estimated ρ and the minimum ρ.

Table C-4. Coverage of confidence intervals by logarithmic Jackknife, Population A

Sample size	Point	Conventional intervals					Logarithmic transform of intervals				
		Trial #1	Trial #2	Trial #3	Trial #4	Average	Trial #1	Trial #2	Trial #3	Trial #4	Average
2400/360	<.025	.02	.00	.00	.01	.0075	.02	.00	.02	.01	.0125
	<.05	.03	.00	.02	.01	.0150	.04	.01	.05	.04	.0350
	Between	.84	.91	.88	.92	.8875	.88	.92	.85	.90	.8875
	>.95	.13	.09	.10	.07	.0975	.08	.07	.10	.06	.0775
	>.975	.06	.06	.07	.05	.0600	.04	.03	.05	.02	.0350
1200/360	<.025	.02	.02	.01	.01	.0150	.04	.02	.04	.03	.0325
	<.05	.05	.03	.04	.03	.0375	.07	.03	.05	.04	.0475
	Between	.87	.89	.88	.90	.8850	.87	.90	.88	.91	.8900
	>.95	.08	.08	.08	.07	.0775	.06	.07	.07	.05	.0625
	>.975	.06	.05	.06	.03	.0500	.05	.02	.04	.03	.0350
880/260	<.025	.01	.00	.01	.03	.0125	.01	.00	.03	.07	.0275
	<.05	.01	.00	.04	.09	.0350	.04	.05	.08	.10	.0675
	Between	.87	.93	.91	.84	.8875	.85	.90	.87	.85	.8675
	>.95	.12	.07	.05	.07	.0775	.11	.05	.05	.05	.0650
	>.975	.11	.05	.05	.06	.0675	.06	.04	.04	.02	.0400
350/160	<.025	.00	.01	.01	.01	.0075	.02	.01	.04	.04	.0275
	<.05	.02	.02	.04	.04	.0300	.04	.03	.06	.08	.0525
	Between	.90	.85	.86	.85	.8650	.91	.87	.89	.85	.8800
	>.95	.08	.13	.10	.11	.1050	.05	.10	.05	.07	.0675
	>.975	.05	.10	.06	.08	.0725	.01	.04	.02	.03	.0250

Note: Each trial used 100 repetitions, and each repetition used 45 replicates.

Figure C-1. Distribution of estimated standard error

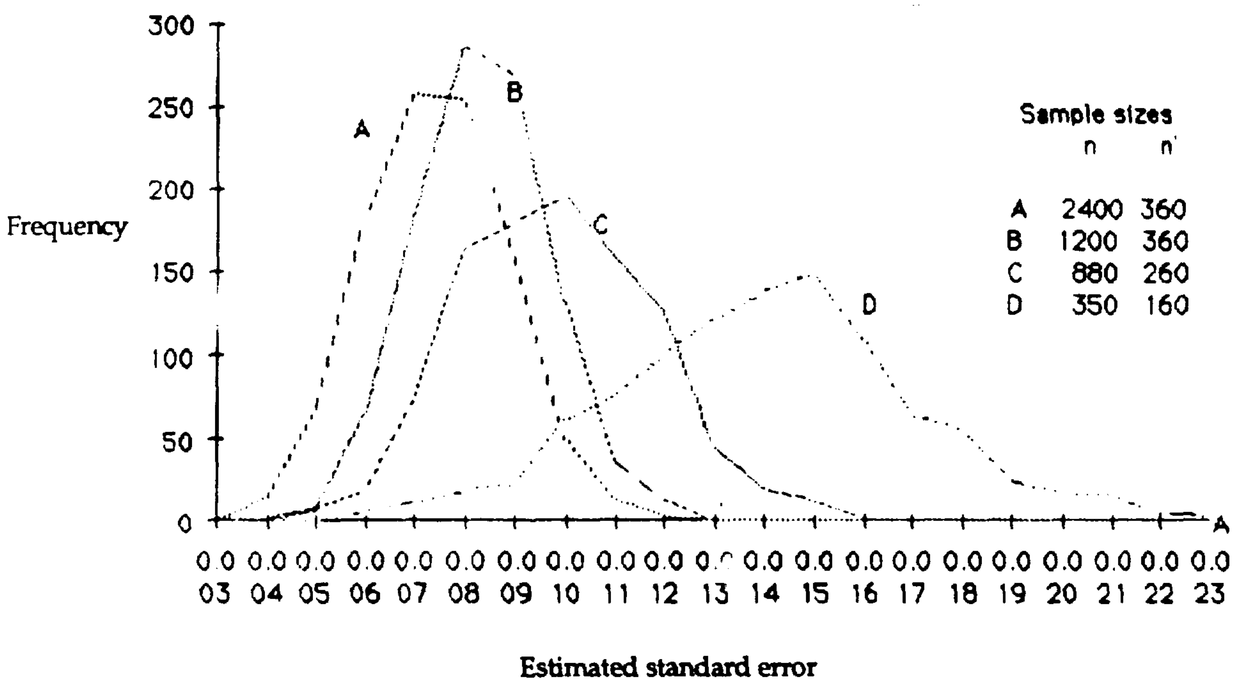

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Figure C-2A. Scatterplot for Population A - sample size 1

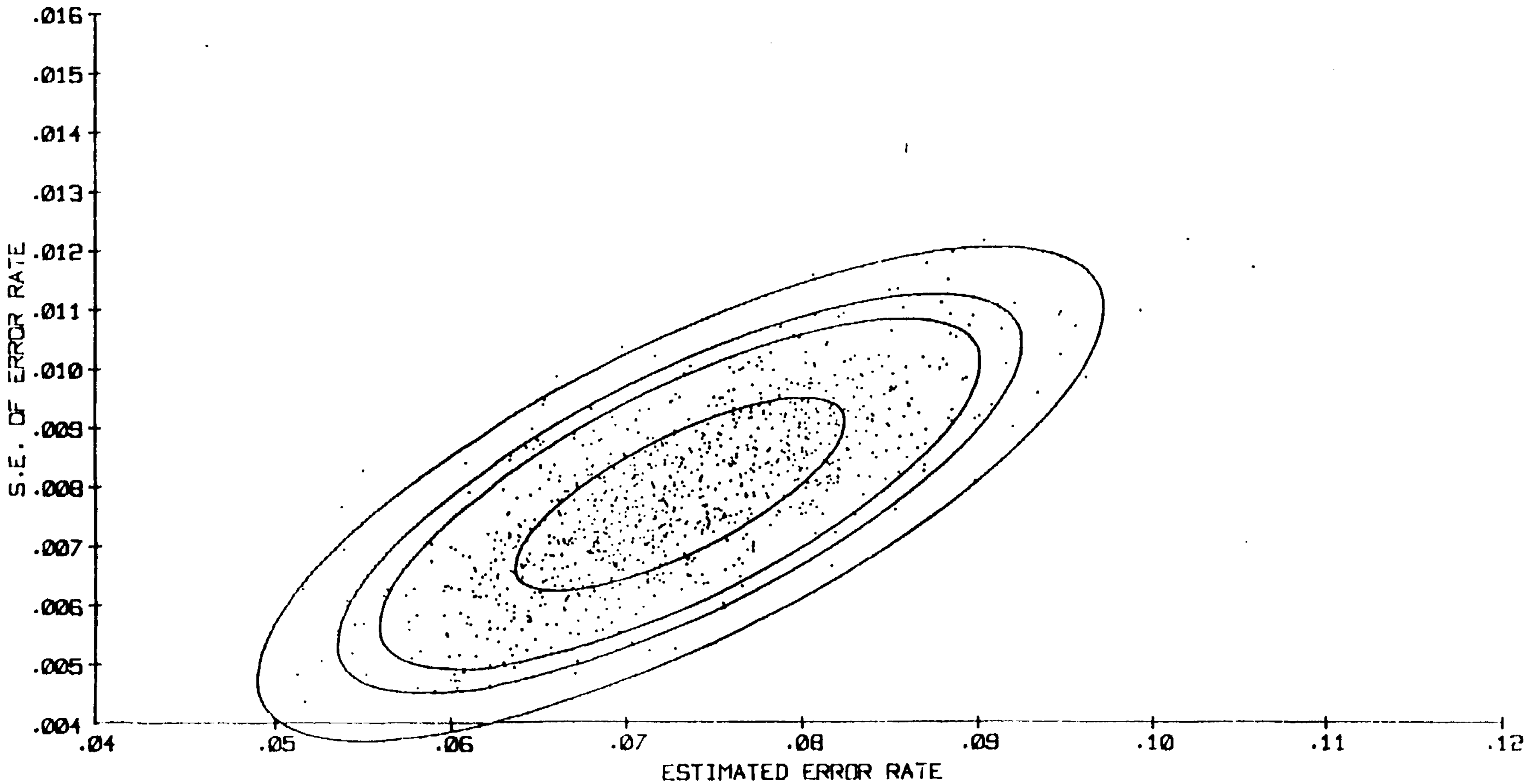

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Figure C-2B. Scatterplot for Population A sample size 2

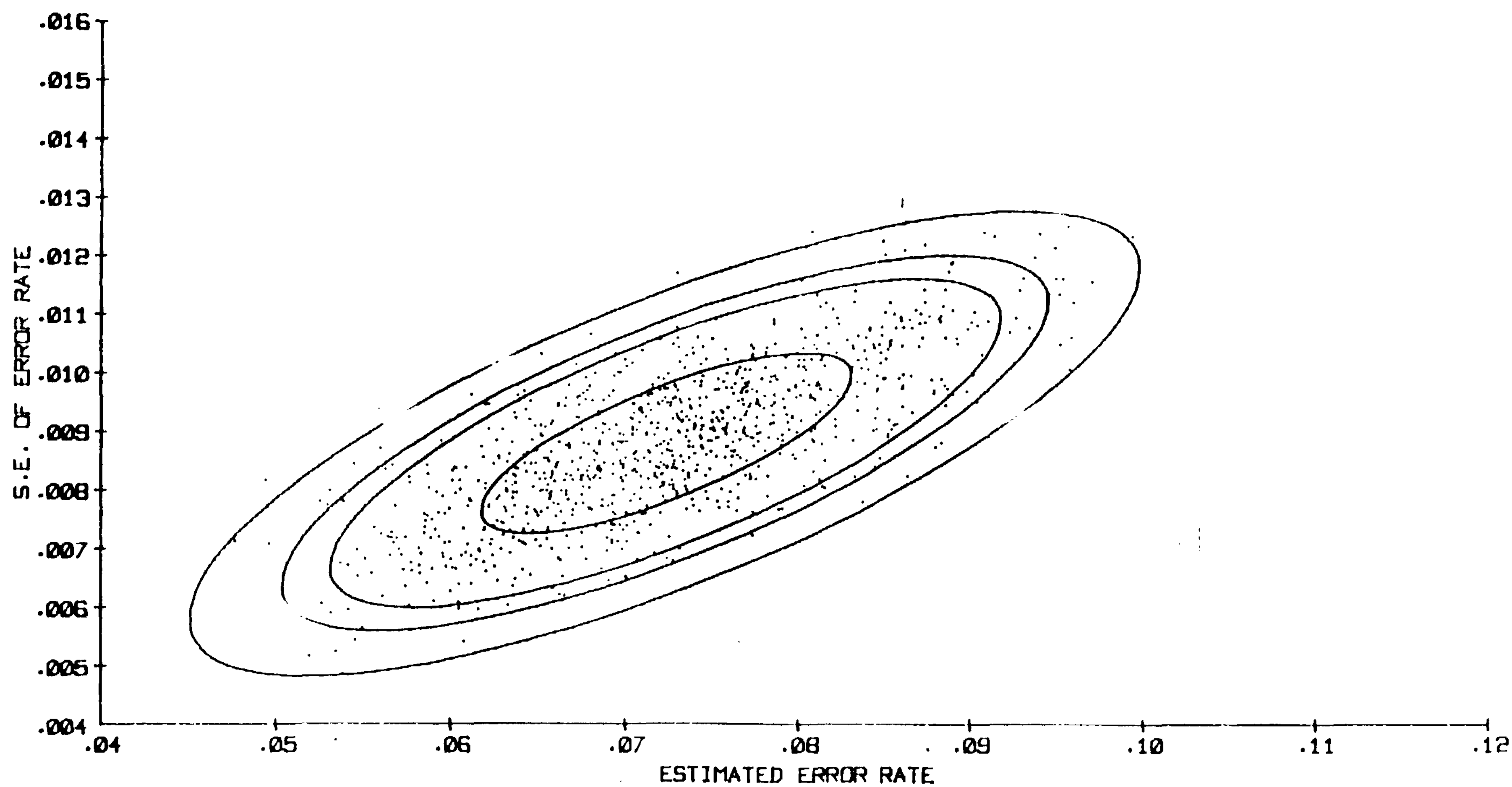

Figure C-2C. Scatterplot for Population A - sample size 3

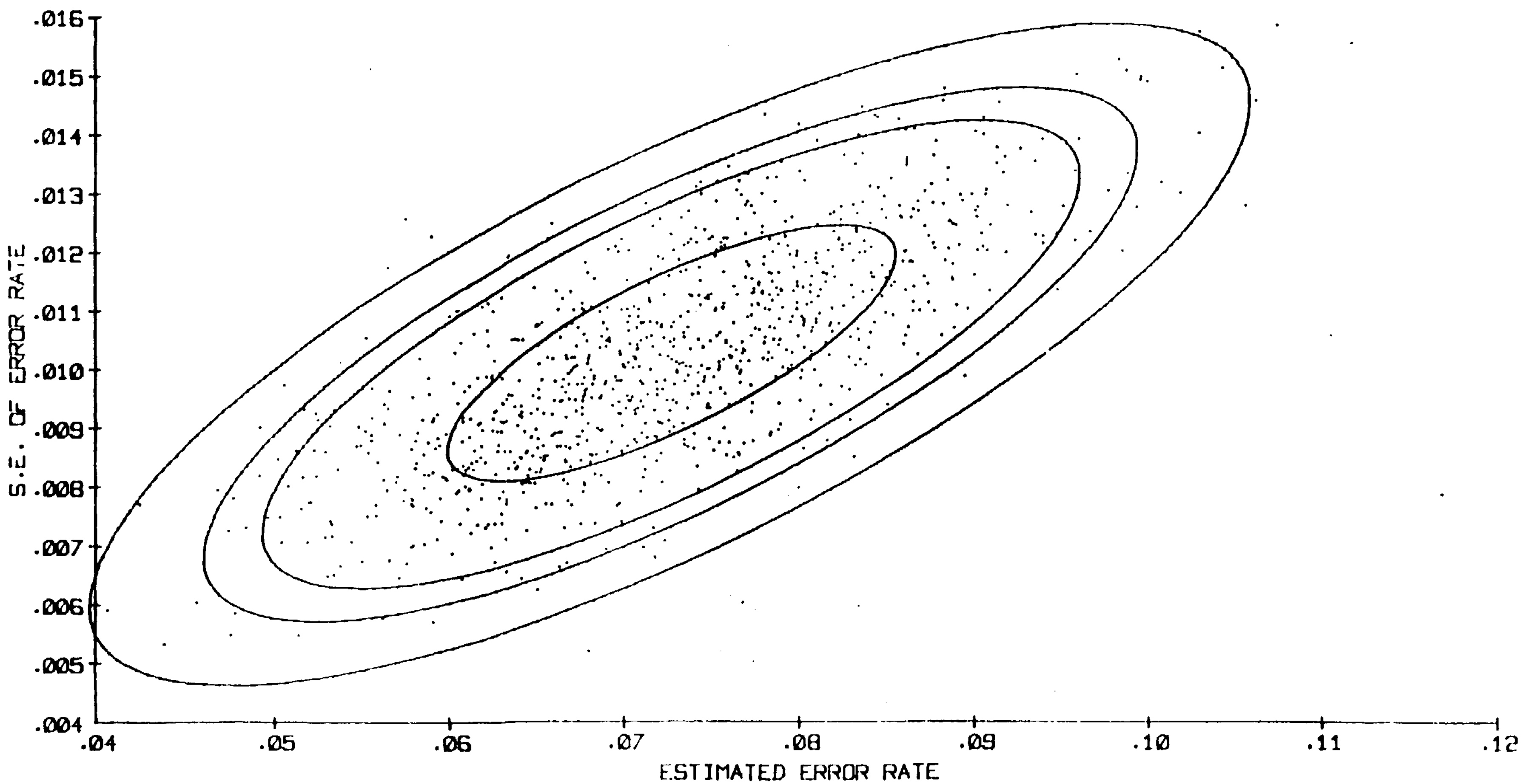

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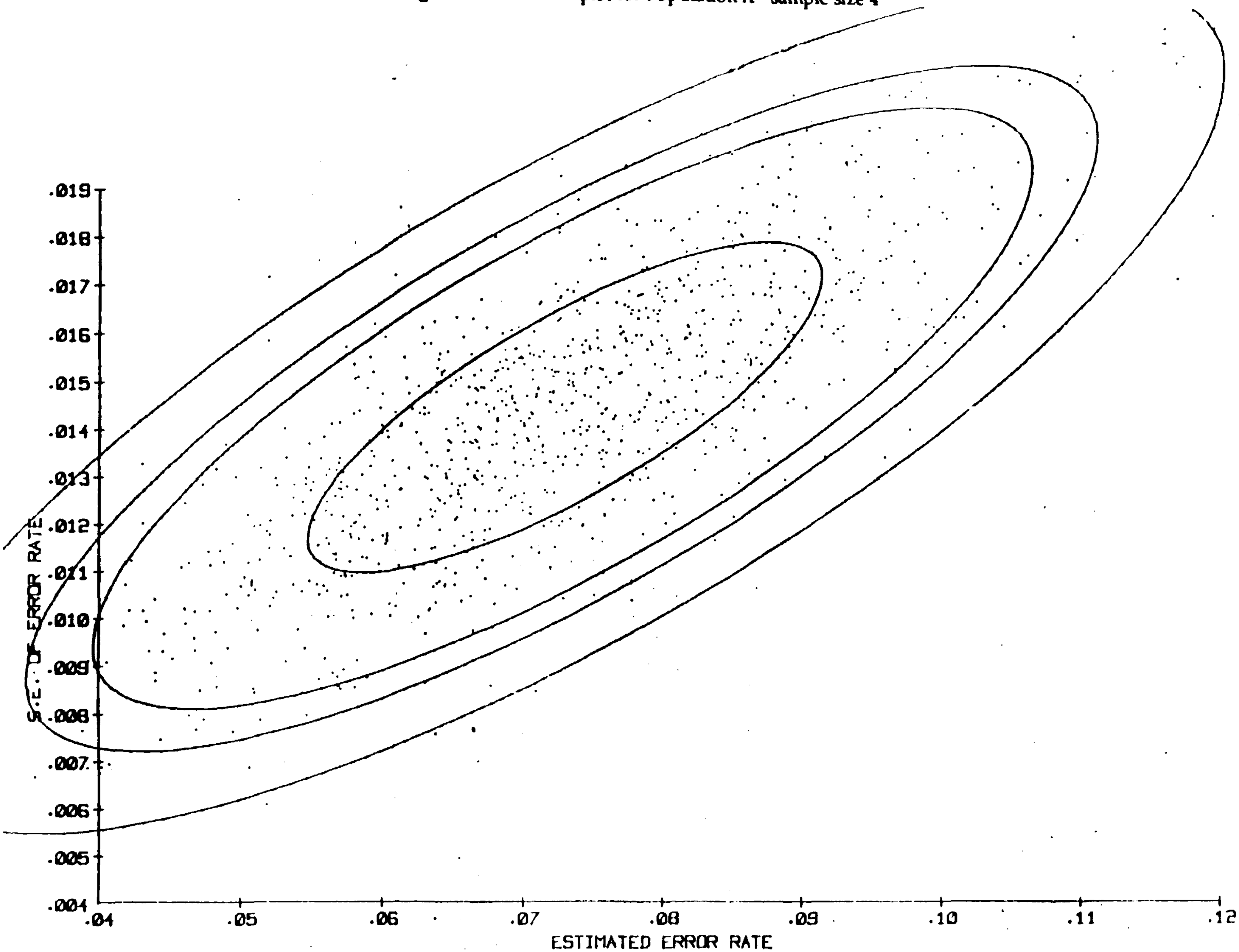
Figure C-2D. Scatterplot for Population A - sample size 4



TECHNICAL NOTE FOR APPENDIX C:

A Note on Confidence Intervals

If a simple random sample of size n is drawn from a normal distribution, the mean of the population may be estimated by

$$\bar{x} = \Sigma x_i / n$$

and the variance of $\bar{x}$ by

$$s_{\bar{x}}^2 = \Sigma(x_i - \bar{x})^2 / (n-1)\, n\,.$$

If $\bar{X}$ denotes the population mean, the statistic $(\bar{x} - \bar{X})/s_{\bar{x}}$ has the Student t distribution so that a confidence interval with confidence coefficient α is given by

$$\bar{x} \pm t(\alpha) s_{\bar{x}}$$

where $t(\alpha)$ is taken from the Student t distribution or from the normal distribution if n is large (say $n>30$).

Even when the conditions given above are not satisfied, the confidence interval is often estimated in the same way, on the assumption that since the distribution of $\bar{x}$ is approximately normal for a large sample, the procedure ensures that the probability that the interval will cover the population mean $\bar{X}$ is approximately α. It is often assumed that the probability that $\bar{X}$ is below (or above) the interval is approximately $(1-\alpha)/2$. The fact, however, is that for samples drawn from skewed distributions the statistics $\bar{x}$ and $s_{\bar{x}}$ are correlated and consequently the probability that $\bar{X}$ is below the interval is not necessarily equal to the probability that $\bar{X}$ is above the interval. Actually, in sampling from skewed distributions, the joint distribution of $\bar{x}$ and $s_{\bar{x}}$ may approach normality reasonably closely for samples of moderate size, but $\bar{x}$ and $s_{\bar{x}}$ remain correlated, and the correlation remains about

the same as sample size increases. Also, the variance of $s_{\bar{x}}$ may be much greater than if sampling from a normal distribution. We evaluate the probabilities associated with 90 percent and 95 percent nominal confidence intervals for this case, i.e., $\bar{x}$ and $s_{\bar{x}}$ are jointly normally distributed but correlated and with various possible values of the coefficient of correlation depending on the skewness of the distribution from which the sample was drawn.

Suppose that a variable u has the normal distribution with mean μ and variance σ^2, and that a variable s has a normal distribution with mean σ and variance τ^2, and that the correlation of u and s is ρ. Let k be a constant and define the upper and lower bounds of a confidence interval by

$$\xi = u \pm ks.$$

The variable ξ is normally distributed, with

$$E(\xi) = \mu \pm k\sigma$$

$$\begin{aligned} Var(\xi) &= Var(u) + k^2 Var(s) \pm 2k\, Cov(u,s) \\ &= \sigma^2 + k^2\tau^2 \pm 2k\rho\sigma\tau \\ &= V^2, \text{ say.} \end{aligned}$$

We wish to evaluate

$$Prob\,(\xi \leq \mu) = (V\sqrt{2\pi})^{-1} \int_{-\infty}^{\mu} \exp\{-(x-\mu\mp k\sigma)^2/2V^2\}\, dx\,.$$

Let

$$y = (x-\mu\mp k\sigma)/V$$

so that

$$x = Vy + \mu \mp k\sigma$$

$$dx = V\,dy$$

and

$$\text{Prob}\,(\xi \le \mu) = (\sqrt{2\pi})^{-1} \int_{-\infty}^{\mp k\sigma/V} \exp(-y^2/2)\,dy\,.$$

We may define

$$z^2 = \frac{V^2}{\sigma^2} = 1 + k^2\left(\frac{\tau}{\sigma}\right)^2 \pm 2k\rho\left(\frac{\tau}{\sigma}\right)$$

so that we may write

$$\text{Prob}\,(\xi \le \mu) = (\sqrt{2\pi})^{-1} \int_{-\infty}^{\mp k/z} \exp(-y^2/2)\,dy\,.$$

Note that τ/σ is the coefficient of variation of s.

The probability that the lower bound of the confidence interval is greater than μ is thus

$$1 - (\sqrt{2\pi})^{-1} \int_{-\infty}^{k/z} \exp(-y^2/2)\,dy$$

and the probability that the upper bound is less than μ is

$$(\sqrt{2\pi})^{-1} \int_{-\infty}^{-k/z} \exp(-y^2/2)\,dy\,.$$

We may call these the coverage probabilities of the lower and upper "tails" of the confidence interval, respectively.

In Table C-5 we show the values of these probabilities for the nominal 95 percent confidence interval (in which case one takes k = 1.96) and for the nominal 90 percent confidence interval (in which case one takes k = 1.645). The computations are shown for various combinations of ρ (in the column headed

"Rho") and τ/σ (in the column headed "CV(s)", the coefficient of variation of the estimated standard error). The coverage probability of the confidence interval itself is simply the complement of the sum of the coverage probabilities of the tails. In each of the columns headed "Bias" we show the difference between the nominal probability and the actual probability. Note that this follows the statistical convention of showing the estimate (taken to be the nominal probability) minus the value being estimated (taken to be the true probability of the tail).

To illustrate, consider a case in which $\rho = .7$ and CV(s) = .1. For a nominal 95 percent confidence interval, the probability that the value being estimated is in the lower tail (i.e., the lower bound is greater than the true value) is .0125 and the probability that the value being estimated is in the upper tail (i.e., the upper bound is less than the true value) is .0436. Since the nominal probabilities are both .025, the biases are, respectively, .025 – .0125 = .0125 and .025 – .0436 = –.0186.

The relevance of this discussion to the AFDC-QC sample estimates is that the estimated error rate $\hat{R}$ and the estimated standard error $s_{\hat{R}}$ are approximately jointly normally distributed, but with positive correlations (these positive correlations are essentially constant for all sample sizes from a given population). Thus, $\hat{R}$ and $s_{\hat{R}}$ are (approximately) examples of the variables u and s in the above analysis. The coverage probabilities read from Table C-5 are reasonably consistent with those estimated from simulated sampling from the test populations as displayed in Table 2-4, for the estimated values of ρ and the coefficient of variation $V_{s_{\hat{R}}}$ given in Table C-1. The tail probabilities of the tails of the nominal confidence intervals, as given by simulated sampling from the test populations with various sample sizes, are compared in Table C-6.

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Table C-5. Bias of nominal coverage probabilities, for samples from a skewed distribution*

Rho	CV(s)	95% Confidence Intervals						90% Confidence Intervals					
		Lower tail		Interval		Upper tail		Lower tail		Interval		Upper tail	
		Prob.	Bias	Prob.	Bias	Prob.	Bias	Prob.	Bias	Prob.	Bias	Prob.	Bias
0.9	0.5	.0000	.0250	.8451	.1049	.1549	-.1299	.0001	.0499	.8226	.0774	.1773	-.1273
0.9	0.4	.0000	.0250	.8701	.0799	.1299	-.1049	.0005	.0495	.8449	.0551	.1546	-.1046
0.9	0.3	.0001	.0249	.8968	.0532	.1031	-.0781	.0029	.0471	.8672	.0328	.1299	-.0799
0.9	0.2	.0017	.0233	.9230	.0270	.0753	-.0503	.0110	.0390	.8854	.0146	.1036	-.0536
0.9	0.1	.0090	.0160	.9428	.0072	.0483	-.0233	.0272	.0228	.8965	.0035	.0763	-.0263
0.9	0.08	.0115	.0135	.9453	.0047	.0432	-.0182	.0313	.0187	.8978	.0022	.0709	-.0209
0.9	0.06	.0143	.0107	.9474	.0026	.0383	-.0133	.0357	.0143	.8988	.0012	.0656	-.0156
0.9	0.04	.0175	.0075	.9488	.0012	.0336	-.0086	.0403	.0097	.8995	.0005	.0603	-.0103
0.9	0.02	.0211	.0039	.9497	.0003	.0292	-.0042	.0450	.0050	.8999	.0001	.0551	-.0051
0.8	0.5	.0009	.0241	.8507	.0993	.1484	-.1234	.0031	.0469	.8261	.0739	.1708	-.1208
0.8	0.4	.0005	.0245	.8758	.0742	.1236	-.0986	.0038	.0462	.8478	.0522	.1484	-.0984
0.8	0.3	.0010	.0240	.9015	.0485	.0975	-.0725	.0073	.0427	.8684	.0316	.1243	-.0743
0.8	0.2	.0035	.0215	.9256	.0244	.0710	-.0460	.0155	.0345	.8854	.0146	.0991	-.0491
0.8	0.1	.0107	.0143	.9434	.0066	.0459	-.0209	.0299	.0201	.8963	.0037	.0738	-.0238
0.8	0.08	.0129	.0121	.9457	.0043	.0413	-.0163	.0335	.0165	.8976	.0024	.0688	-.0188
0.8	0.06	.0155	.0095	.9476	.0024	.0369	-.0119	.0373	.0127	.8987	.0013	.0640	-.0140
0.8	0.04	.0184	.0066	.9489	.0011	.0327	-.0077	.0414	.0086	.8994	.0006	.0592	-.0092
0.8	0.02	.0215	.0035	.9497	.0003	.0287	-.0037	.0456	.0044	.8999	.0001	.0545	-.0045
0.7	0.5	.0053	.0197	.8532	.0968	.1415	-.1165	.0116	.0384	.8244	.0756	.1640	-.1140
0.7	0.4	.0032	.0218	.8798	.0702	.1170	-.0920	.0107	.0393	.8474	.0526	.1418	-.0918
0.7	0.3	.0033	.0217	.9050	.0450	.0916	-.0666	.0135	.0365	.8681	.0319	.1185	-.0685
0.7	0.2	.0059	.0191	.9276	.0224	.0665	-.0415	.0205	.0295	.8850	.0150	.0945	-.0445
0.7	0.1	.0125	.0125	.9440	.0060	.0436	-.0186	.0327	.0173	.8961	.0039	.0712	-.0212
0.7	0.08	.0145	.0105	.9461	.0039	.0394	-.0144	.0358	.0142	.8975	.0025	.0667	-.0167
0.7	0.06	.0167	.0083	.9478	.0022	.0355	-.0105	.0390	.0110	.8986	.0014	.0623	-.0123
0.7	0.04	.0192	.0058	.9490	.0010	.0318	-.0068	.0425	.0075	.8994	.0006	.0581	-.0081
0.7	0.02	.0220	.0030	.9498	.0002	.0283	-.0033	.0462	.0038	.8999	.0001	.0540	-.0040

*(Based on a model in which $\bar{x}$ and $s_{\bar{x}}$ have a bivariate normal distribution with correlation ρ.)

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Table C-5. Bias of nominal coverage probabilities, for samples from a skewed distribution* (continued)

Rho	CV(s)	95% Confidence Intervals						90% Confidence Intervals					
		Lower tail		Interval		Upper tail		Lower tail		Interval		Upper tail	
		Prob.	Bias	Prob.	Bias	Prob.	Bias	Prob.	Bias	Prob.	Bias	Prob.	Bias
0.6	0.5	.0134	.0116	.8523	.0977	.1342	-.1092	.0238	.0262	.8195	.0805	.1567	-.1067
0.6	0.4	.0085	.0165	.8814	.0686	.1101	-.0851	.0201	.0299	.8449	.0551	.1349	-.0849
0.6	0.3	.0071	.0179	.9073	.0427	.0856	-.0606	.0208	.0292	.8669	.0331	.1124	-.0624
0.6	0.2	.0089	.0161	.9291	.0209	.0620	-.0370	.0257	.0243	.8844	.0156	.0898	-.0398
0.6	0.1	.0144	.0106	.9444	.0056	.0412	-.0162	.0355	.0145	.8960	.0040	.0686	-.0186
0.6	0.08	.0161	.0089	.9464	.0036	.0376	-.0126	.0380	.0120	.8974	.0026	.0646	-.0146
0.6	0.06	.0179	.0071	.9480	.0020	.0341	-.0091	.0407	.0093	.8986	.0014	.0607	-.0107
0.6	0.04	.0201	.0049	.9491	.0009	.0308	-.0058	.0436	.0064	.8994	.0006	.0570	-.0070
0.6	0.02	.0224	.0026	.9498	.0002	.0278	-.0028	.0467	.0033	.8999	.0001	.0534	-.0034
0.5	0.5	.0239	.0011	.8496	.1004	.1265	-.1015	.0375	.0125	.8134	.0866	.1490	-.0990
0.5	0.4	.0158	.0092	.8814	.0686	.1028	-.0778	.0308	.0192	.8415	.0585	.1276	-.0776
0.5	0.3	.0122	.0128	.9085	.0415	.0793	-.0543	.0288	.0212	.8653	.0347	.1060	-.0560
0.5	0.2	.0124	.0126	.9302	.0198	.0575	-.0325	.0312	.0188	.8838	.0162	.0850	-.0350
0.5	0.1	.0164	.0086	.9448	.0052	.0389	-.0139	.0383	.0117	.8958	.0042	.0659	-.0159
0.5	0.08	.0177	.0073	.9466	.0034	.0357	-.0107	.0402	.0098	.8973	.0027	.0624	-.0124
0.5	0.06	.0192	.0058	.9481	.0019	.0327	-.0077	.0424	.0076	.8985	.0015	.0591	-.0091
0.5	0.04	.0209	.0041	.9492	.0008	.0299	-.0049	.0448	.0052	.8994	.0006	.0559	-.0059
0.5	0.02	.0229	.0021	.9498	.0002	.0273	-.0023	.0473	.0027	.8999	.0001	.0529	-.0029
0.4	0.5	.0354	-.0104	.8462	.1038	.1184	-.0934	.0516	-.0016	.8076	.0924	.1408	-.0908
0.4	0.4	.0243	.0007	.8805	.0695	.0953	-.0703	.0420	.0080	.8380	.0620	.1200	-.0700
0.4	0.3	.0181	.0069	.9090	.0410	.0729	-.0479	.0371	.0129	.8636	.0364	.0994	-.0494
0.4	0.2	.0162	.0088	.9309	.0191	.0528	-.0278	.0368	.0132	.8832	.0168	.0801	-.0301
0.4	0.1	.0184	.0066	.9451	.0049	.0365	-.0115	.0411	.0089	.8957	.0043	.0632	-.0132
0.4	0.08	.0194	.0056	.9468	.0032	.0338	-.0088	.0425	.0075	.8972	.0028	.0603	-.0103
0.4	0.06	.0205	.0045	.9482	.0018	.0313	-.0063	.0441	.0059	.8985	.0015	.0574	-.0074
0.4	0.04	.0218	.0032	.9492	.0008	.0290	-.0040	.0459	.0041	.8993	.0007	.0548	-.0048
0.4	0.02	.0233	.0017	.9498	.0002	.0269	-.0019	.0478	.0022	.8999	.0001	.0523	-.0023

*(Based on a model in which $\bar{x}$ and $s_{\bar{x}}$ have a bivariate normal distribution with correlation ρ.)

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Table C-5. Bias of nominal coverage probabilities, for samples from a skewed distribution* (continued)

Rho	CV(s)	95% Confidence Intervals						90% Confidence Intervals					
		Lower tail		Interval		Upper tail		Lower tail		Interval		Upper tail	
		Prob.	Bias	Prob.	Bias	Prob.	Bias	Prob.	Bias	Prob.	Bias	Prob.	Bias
0.3	0.5	.0472	-.0222	.8431	.1069	.1098	-.0848	.0652	-.0152	.8027	.0973	.1321	-.0821
0.3	0.4	.0335	-.0085	.8792	.0708	.0873	-.0623	.0532	-.0032	.8349	.0651	.1118	-.0618
0.3	0.3	.0246	.0004	.9091	.0409	.0663	-.0413	.0455	.0045	.8620	.0380	.0925	-.0425
0.3	0.2	.0204	.0046	.9314	.0186	.0481	-.0231	.0424	.0076	.8826	.0174	.0750	-.0250
0.3	0.1	.0205	.0045	.9453	.0047	.0342	-.0092	.0439	.0061	.8956	.0044	.0605	-.0105
0.3	0.08	.0211	.0039	.9470	.0030	.0319	-.0069	.0447	.0053	.8972	.0028	.0581	-.0081
0.3	0.06	.0218	.0032	.9483	.0017	.0299	-.0049	.0458	.0042	.8984	.0016	.0558	-.0058
0.3	0.04	.0227	.0023	.9492	.0008	.0281	-.0031	.0470	.0030	.8993	.0007	.0537	-.0037
0.3	0.02	.0237	.0013	.9498	.0002	.0264	-.0014	.0484	.0016	.8999	.0001	.0517	-.0017

*(Based on a model in which $\bar{x}$ and $s_{\bar{x}}$ have a bivariate normal distribution with correlation ρ.)

Table C-6. Tail coverages as estimated by simulation and as given by the normal model

			95% Confidence Interval				90% Confidence Interval			
			Lower tail		Upper tail		Lower tail		Upper tail	
Sample size	Rho	CV(s)	Estimated	Modeled	Estimated	Modeled	Estimated	Modeled	Estimated	Modeled
Population A										
2400/360	0.75	0.18	0.011	0.006	0.053	0.064	0.024	0.020	0.084	0.092
1200/3[illegible]	0.75	0.14	0.006	0.008	0.059	0.054	0.028	0.025	0.097	0.082
880/260	0.76	0.18	0.010	0.005	0.066	0.064	0.028	0.020	0.100	0.092
350/160	0.79	0.20	0.013	0.004	0.075	0.071	0.031	0.016	0.102	0.099
Population B										
2400/360	0.66	0.20	0.011	0.007	0.067	0.065	0.032	0.023	0.093	0.093
1200/350	0.62	0.16	0.012	0.010	0.042	0.054	0.030	0.028	0.072	0.082
880/260	0.61	0.18	0.008	0.009	0.055	0.058	0.033	0.027	0.093	0.086
350/160	0.67	0.24	0.017	0.005	0.062	0.075	0.036	0.019	0.096	0.103
Population C										
2400/360	0.68	0.27	0.003	0.004	0.060	0.083	0.014	0.016	0.093	0.110
1200/350	0.66	0.22	0.011	0.006	0.080	0.070	0.021	0.021	0.103	0.097
880/260	0.68	0.26	0.009	0.005	0.084	0.080	0.020	0.017	0.113	0.108
350/160	0.71	0.30	0.007	0.003	0.087	0.092	0.028	0.013	0.120	0.119

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APPENDIX D

RELIABILITY OF LOWER CONFIDENCE BOUNDS

D.1 Variances of Lower Confidence Bounds and Point Estimates Compared

The estimated variances and standard errors of the regression estimate of R and of the lower bound of the confidence interval, based on 1000 independent replicates sampled from each test population, for each of several sample sizes, are shown in Table D-1. In this analysis, the lower confidence bound, L, has been computed at the 95 (or 5) percent nominal confidence level, i.e., $L = \hat{R} - ts_{\hat{R}}$ with $t = 1.645$. From the table, it can be seen that the estimated variances of the lower confidence bounds (s_L^2) vary from about one-third to two-thirds as large as the variances of the estimated payment error rates ($s_{\hat{R}}^2$), depending on the state sample size and the fraction in the Federal subsample. The standard errors of L vary from about 60 to 80 percent of the standard error of $\hat{R}$.

Table D-1. Variances and standard errors of 95 percent lower confidence bounds and of estimated payment error rates, for regression estimator, for three test populations for seven illustrative sample sizes

Sample size			Population A		Population B		Population C	
n	n'	n'/n	$s_L^2/s_{\hat{R}}^2$	$s_L/s_{\hat{R}}$	$s_L^2/s_{\hat{R}}^2$	$s_L/s_{\hat{R}}$	$s_L^2/s_{\hat{R}}^2$	$s_L/s_{\hat{R}}$
2400	360	.15	.65	.81	.69	.83	.60	.77
1200	360	.30	.70	.84	.75	.86	.65	.81
880	260	.30	.65	.81	.73	.85	.61	.78
350	160	.46	.60	.78	.64	.80	.55	.74
1200	180	.15	.40	.64	n/a	n/a	n/a	n/a
500	80	.16	.36	.60	n/a	n/a	n/a	n/a
300	50	.17	.32	.56	n/a	n/a	n/a	n/a

These results are both surprising and interesting. They are far different from what would occur in estimating a mean and computing confidence intervals from a simple random sample from approximately normal distributions. They would also have desirable implications for AFDC if lower confidence bounds were to be used in determining disallowances. The relatively smaller variances of L occur because $\hat{R}$ and $s_{\hat{R}}$ are positively correlated. Consequently, if $\hat{R}$ is high, then $s_{\hat{R}}$ tends also to be high and the computed lower bound is, on the average, lower than it would be if the standard error of $\hat{R}$ were known and used to compute it, and vice versa. On the other hand, in sampling from a normal distribution, the estimated mean and its estimated standard error are uncorrelated and there is no such compensation in the computed lower confidence bound, and the variance of the computed lower confidence bound would be larger than the variance of the mean.

The estimated correlations observed in the sets of 1000 replicates for various sample sizes from the three test populations are summarized in Table C-1 in Appendix C, and are seen to be quite high (of the order of .6 to .8). They vary trivially with sample size, and this variation apparently is due primarily to sampling variability.

To provide additional insight, since the nominal 95 percent lower confidence bound is

$$L = \hat{R} - 1.645\, s_{\hat{R}},$$

it follows that the variance of L is

$$\sigma_L^2 = \sigma_{\hat{R}}^2 + (1.645)^2\, \sigma_{s_{\hat{R}}}^2 - 2(1645)\, \rho\, \sigma_{\hat{R}}\, \sigma_{s_{\hat{R}}}$$

where ρ is the correlation of L and s_R.

The first term in σ_L^2 is the variance of $\hat{R}$; the second term is the contribution from the variance of the estimated $s_{s_{\hat{R}}}$, the standard error of $\hat{R}$; and the third term is determined by ρ, the correlation of $\hat{R}$ and $s_{\hat{R}}$. Some estimates of σ_R^2 and $\sigma_{s_{\hat{R}}}^2$ based on the 1000 replicates are given in Tables C-2A, B, and C, and are summarized in Table C-1 in Appendix C. Estimates of ρ are also given in Table C-1. The variance of the lower confidence bound for the regression estimator can be obtained by making the appropriate substitutions in the above equation for σ_L^2. The results agree closely with the values given in Table D-1, which were obtained by computing the variance of L directly from the 1000 replicates.

The implication of these results, as stated earlier (Section 2.5.2), is that the lower confidence bound computed by use of the estimated standard error of $\hat{R}$ from the sample is a substantially more stable and better way to compute the lower confidence bound than would be obtained if the unknown true value of the standard error were in fact known and used in computing the lower confidence limit.

D.2 Use of Minimum Correlation in Computing Lower Confidence Bound to Control Possible Lower Quality of State QC

It has been recognized at OFA, and is a source of concern, that if a lower confidence bound is used in computing disallowances, a state could achieve a considerably lower average disallowance simply by doing a lower-quality QC job, and thereby yielding a lower correlation between the Federal review results and the state QC results. This effect can be seen by examining the role of r (the correlation) in Equation (3), Chapter 1. While it may or may not be likely that this would occur in practice, there is a concern that it might, since the higher the quality of the work done on QC in a state, the higher the correlation, and, as a consequence, the higher the lower confidence bound and the higher the disallowance.

There is a simple solution to this potential problem. The procedure is to identify those states for which r, the estimated correlation between the state and Federal QC results, is less than r_L, where r_L is, perhaps, the 30th percentile of the state estimates of r for the prior year; that is, r_L is the value such that 30 percent of the observed state correlations of state and Federal payment errors in the prior year are below r_L, and 70 percent are above. An acceptable variant of this procedure is to substitute a constant value for r_L that would approximate the 30 percent rule. The constant can be chosen based on recent prior experience. We would expect that for many or most states for which the estimated correlation is below r_L, the low correlation will occur primarily because of sampling variability. The procedure is to substitute r_L for r in Equation (3) of Chapter 1 in estimating the variance of $\hat{R}$ whenever r is less than r_L. The principal gain from this procedure is that it removes or reduces any gain that could result if a state did poorer-quality QC work in order to reduce disallowances. An additional minor advantage is that it slightly reduces the variance of the lower confidence bounds, at the cost of a slight downward bias in the variance estimate.

We illustrate the application of this procedure as follows. Suppose the "30 percent" rule is adopted, and that $r_L = .80$ is the 30th percentile of the state correlations for the prior year. Suppose that for a particular state n' = 360 and n = 2400, and the observed correlation is .50. This relatively low correlation might arise either because the state QC reviewers have done poor work (whether purposefully or not), or because of random variation, or some of both. The ratio of the computed standard error of $\hat{R}$ with .50 substituted for r in Equation (3) to the standard error if .80 is substituted is 1.31. Thus, the use of the standard error computed with $r_L = .80$ substituted for r will substantially raise the lower confidence limit.

Table D-2 shows the distribution of the estimated state correlations for each fiscal year from 1981 to 1984 for the 44 states that did not treat the QC samples as stratified samples in making sample estimates in any of the four years. Figure D-1 shows the cumulative distribution of the correlations for each year for the same 44 states. Figures D-2A, D-2B, and D-2C show the cumulative distribution of the estimated correlation, based on the 1000 independent samples from each of the Test

Populations A, B, and C, respectively, for each sample size. It will be noted that in each of these three figures the two distributions for which the Federal subsample size n' is 360 are nearly indistinguishable.

Figures D-3A, D-3B, and D-3C illustrate for Test Populations A, B, and C the reductions in variance that result from the application of the 30th percentile rule where all correlations are estimated from samples of the same population. Note that in these figures the curves based on the estimated and the minimum correlations are almost indistinguishable. When they overlap, only one is shown.

We note that whether or not the rule of substituting r_L for r is applied in computing the standard error of $\hat{R}$ for a state, the value of $\hat{R}$ is based entirely on the sample for the state, and the computation of $\hat{R}$ is unaffected by the substitution of r_L for r. Also, while the use of the minimum correlation rule makes a substantial difference in the variance estimates for individual states for which the estimated correlation is low, it only moderately reduces the estimated variance over all possible samples that could be drawn. This is clearly illustrated by Figures D-3A, D-3B, and D-3C.

We note another important point in connection with the possible use of lower confidence bounds for assessing disallowances. This is that the lower confidence bound, and consequently the expected disallowance, would average lower for a relatively small than for a relatively large size of QC sample. This could create an incentive for a state with a relatively high error rate to use smaller QC samples just to reduce the potential for disallowances, even though it would be undesirable from the point of view of corrective action and other uses of the quality control sample, as well as from the Federal goal of achieving an acceptable return from disallowances. Consequently, it would be necessary, if a lower confidence bound approach were adopted, to specify minimum sample sizes, and these minima should not be so small as to unreasonably lower the expected lower confidence bounds. Of course, relatively larger samples will also better serve the basic role for which QC was created, i.e., providing guidance for improved AFDC design, and for taking corrective action to improve administration. This issue of desired (optimum) size of QC sample for computing disallowance is briefly considered in Section 3.4 and in Appendix G.

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Table D-2. Distribution of states by the estimated correlation between state and Federal findings for fiscal years 1981-1984, for 44 states

Estimated correlation	Fiscal years				All years
	1981	1982	1983	1984	
.40 - .49	0	4	0	0	4
.50 - .59	7	3	1	0	11
.60 - .69	3	2	3	2	10
.70 - .74	2	3	4	0	9
.75 - .79	5	6	5	4	20
.80 - .84	6	7	5	5	23
.85 - .89	3	7	5	9	24
.90 - .94	7	5	11	12	35
.95 - .99	9	5	10	10	34
1.00	2	2	0	2	6
Totals	44	44	44	44	176
Median	.846	.837	.881	.905	.875
30th percentile	.760	.780	.782	.870	.791

Note: The correlations are tallied only for the states that did not use stratified samples.

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Figure D-1. Cumulative distribution of estimated correlation for 44 states

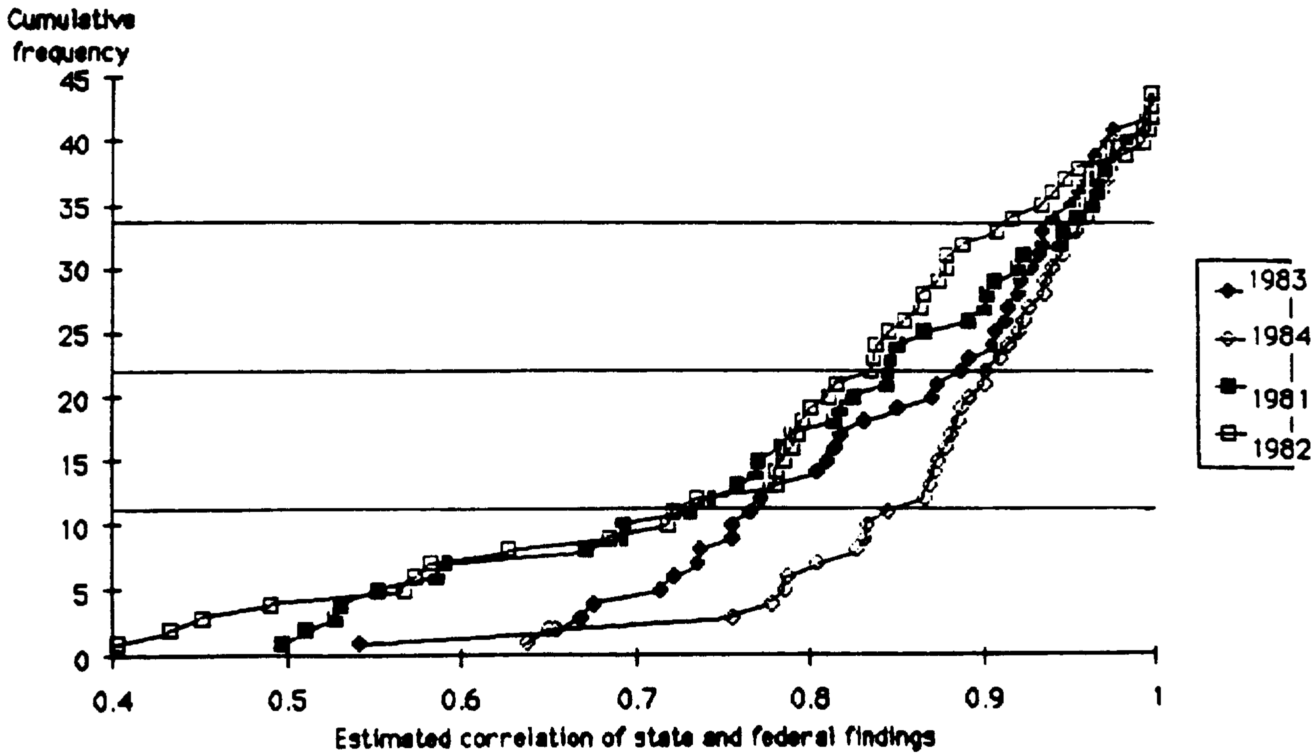

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Figure D-2A. Cumulative distribution of the estimated correlation, Population A

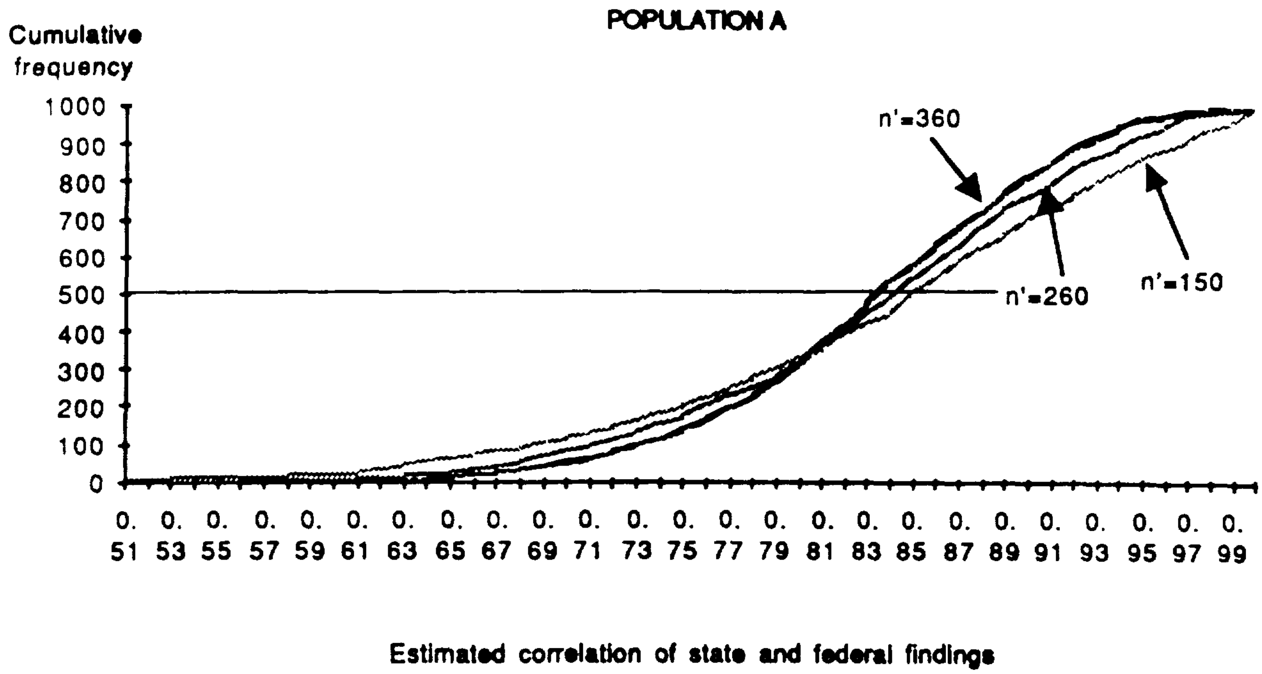

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Figure D-2B. Cumulative distribution of the estimated correlation, Population B

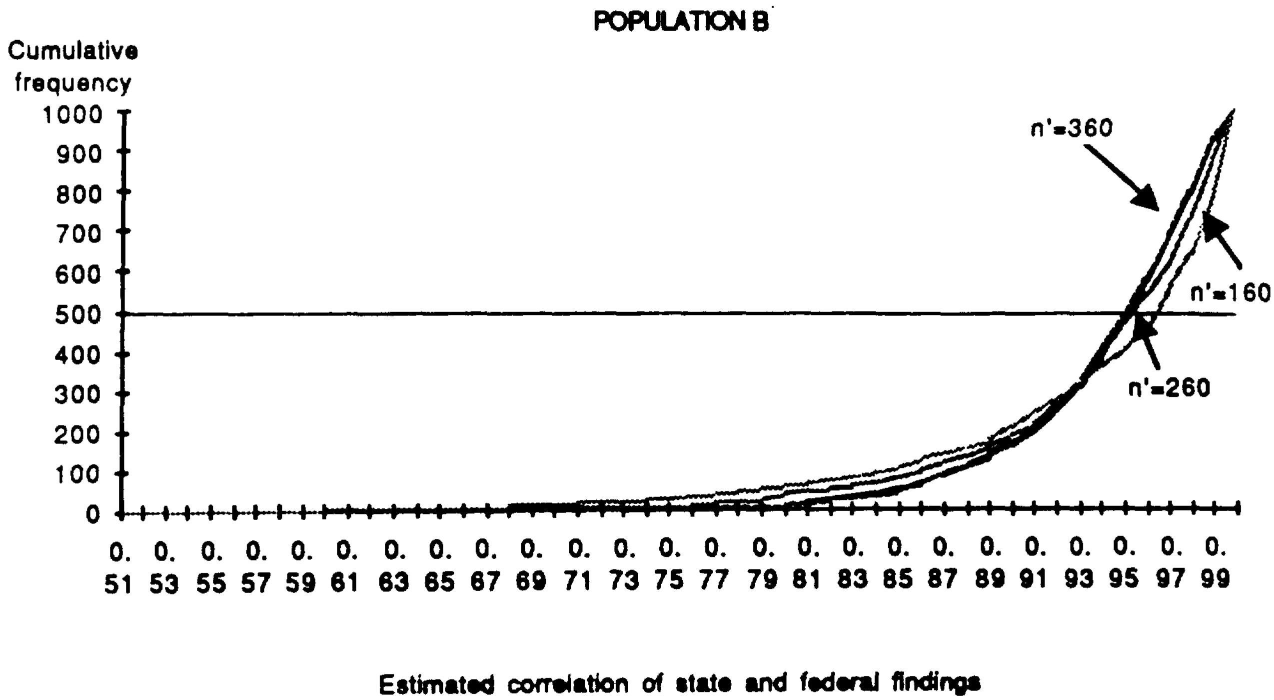

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Figure D-2C. Cumulative distribution of the estimated correlation, Population C

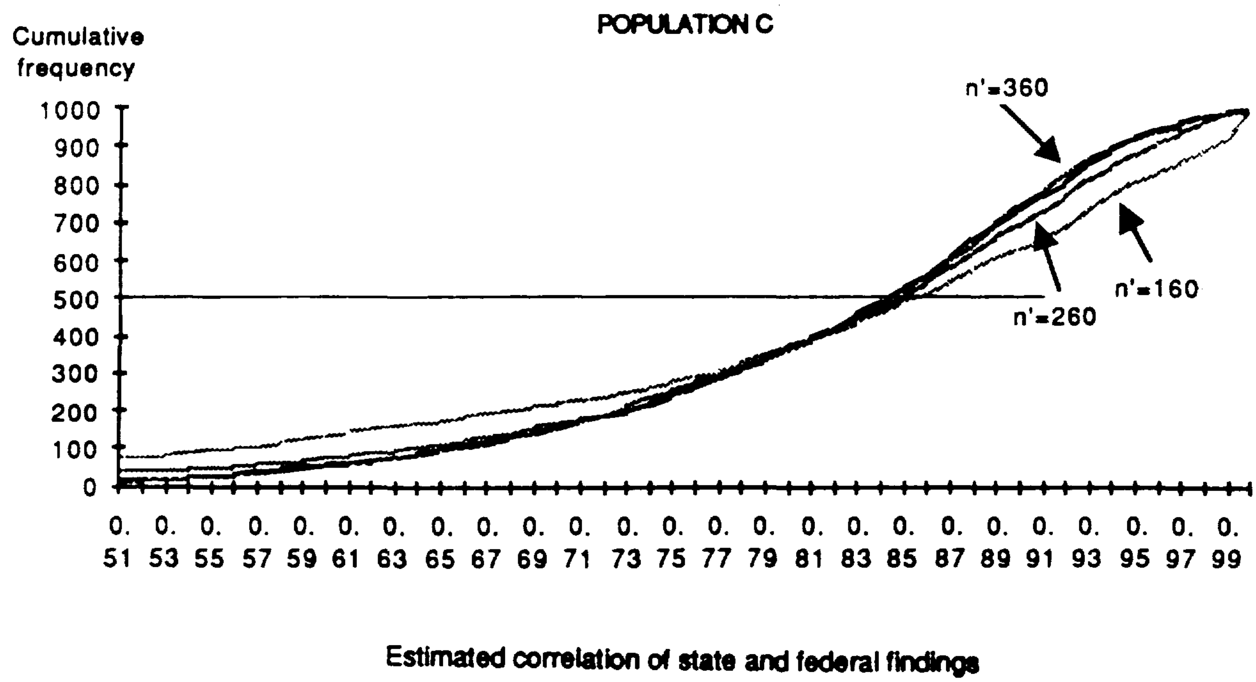

Figure D-3A. Cumulative distribution of the nominal 95 percent lower confidence bounds of the payment error rate using (A) the estimated correlation, and (B) the minimum correlation rule, in the regression estimate of variance, for Population A (based on independent simulations of 1000 samples)

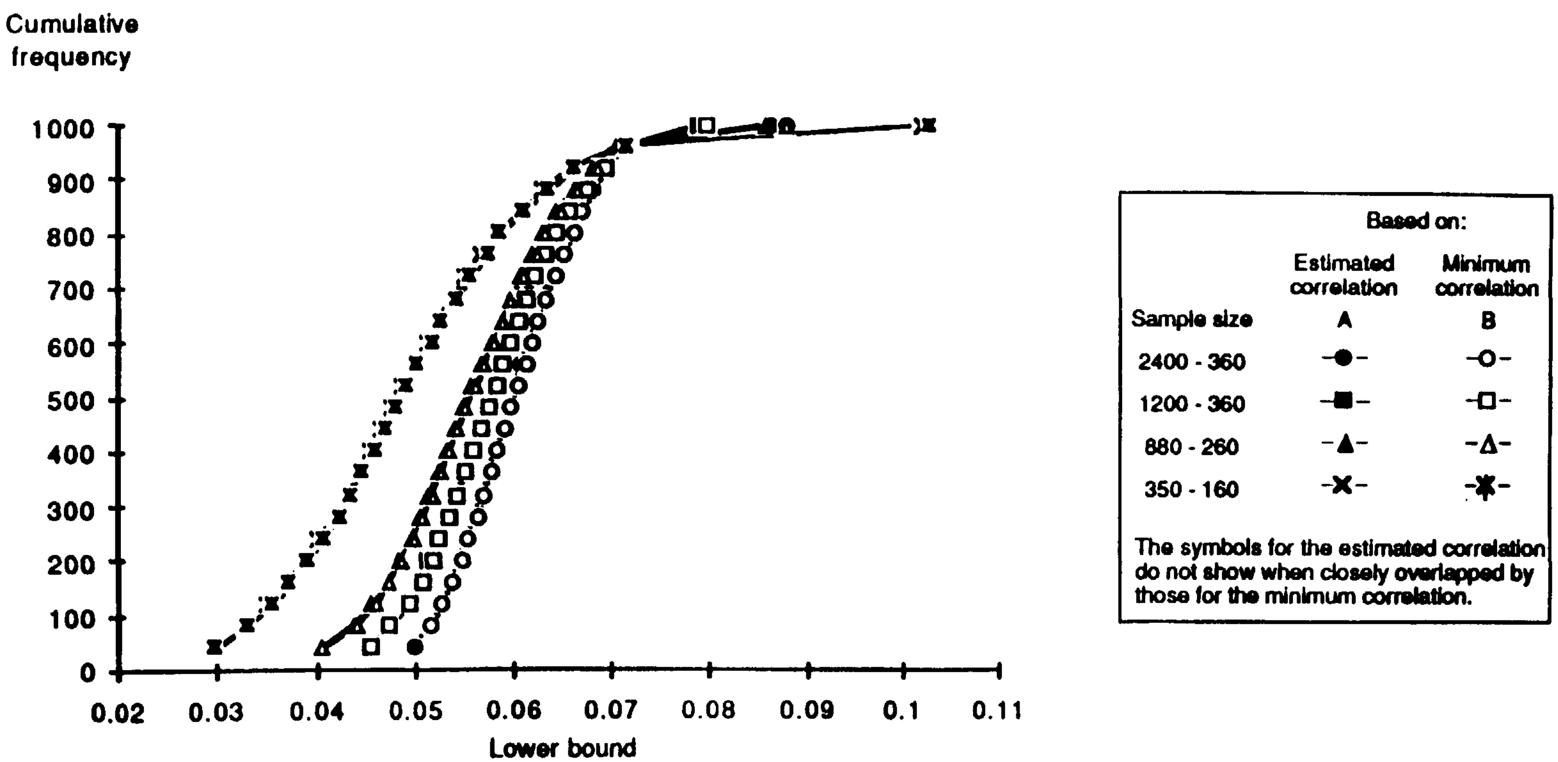

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Figure D-3B. Cumulative distribution of the nominal 95 percent lower c[illegible]nce bou[illegible] of the payment error rate using (A) the estimated correlation, and (B) the minimum correlation rule, in the regression estimate of variance, for Population B (based on independent simulations of 1000 samples)

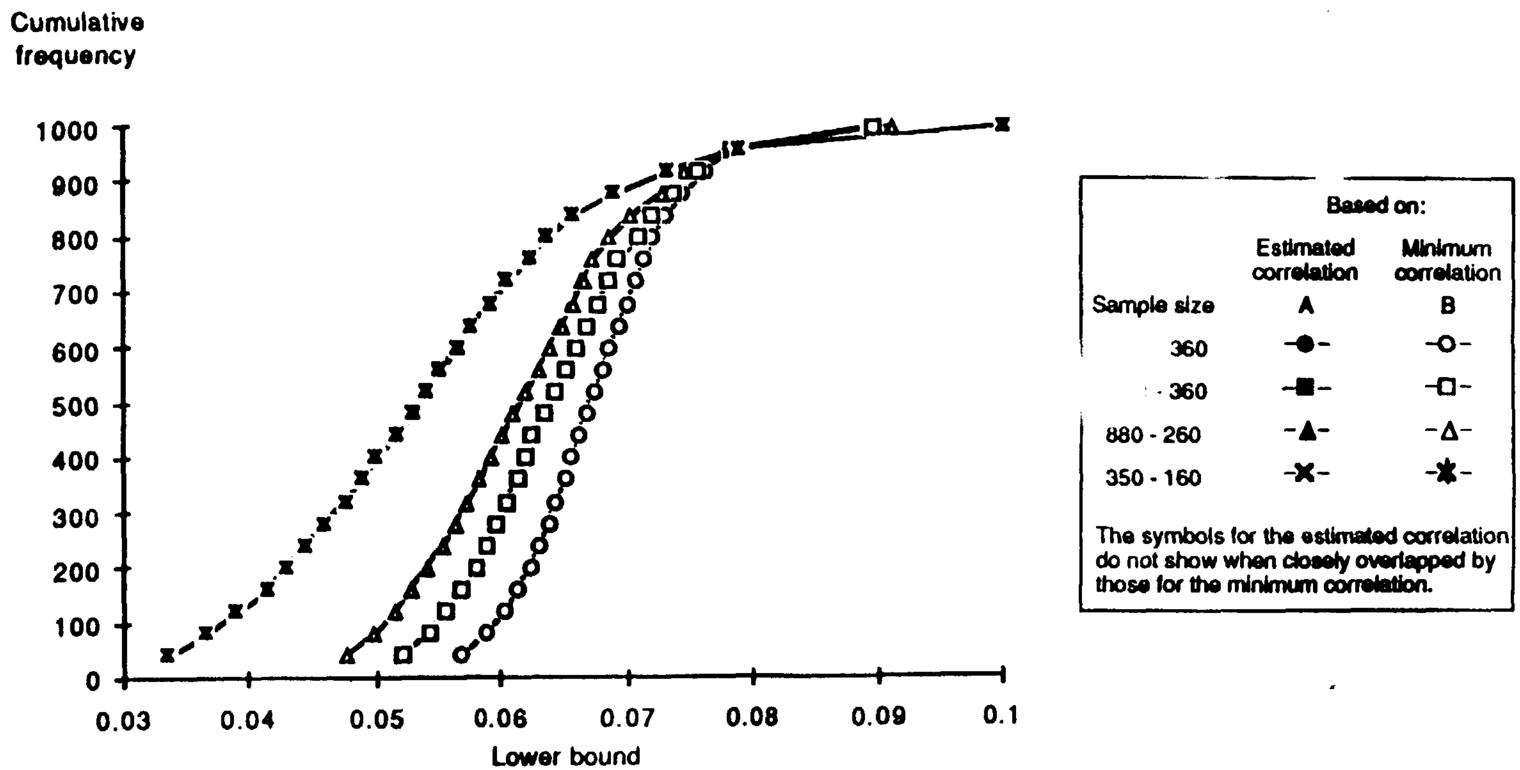

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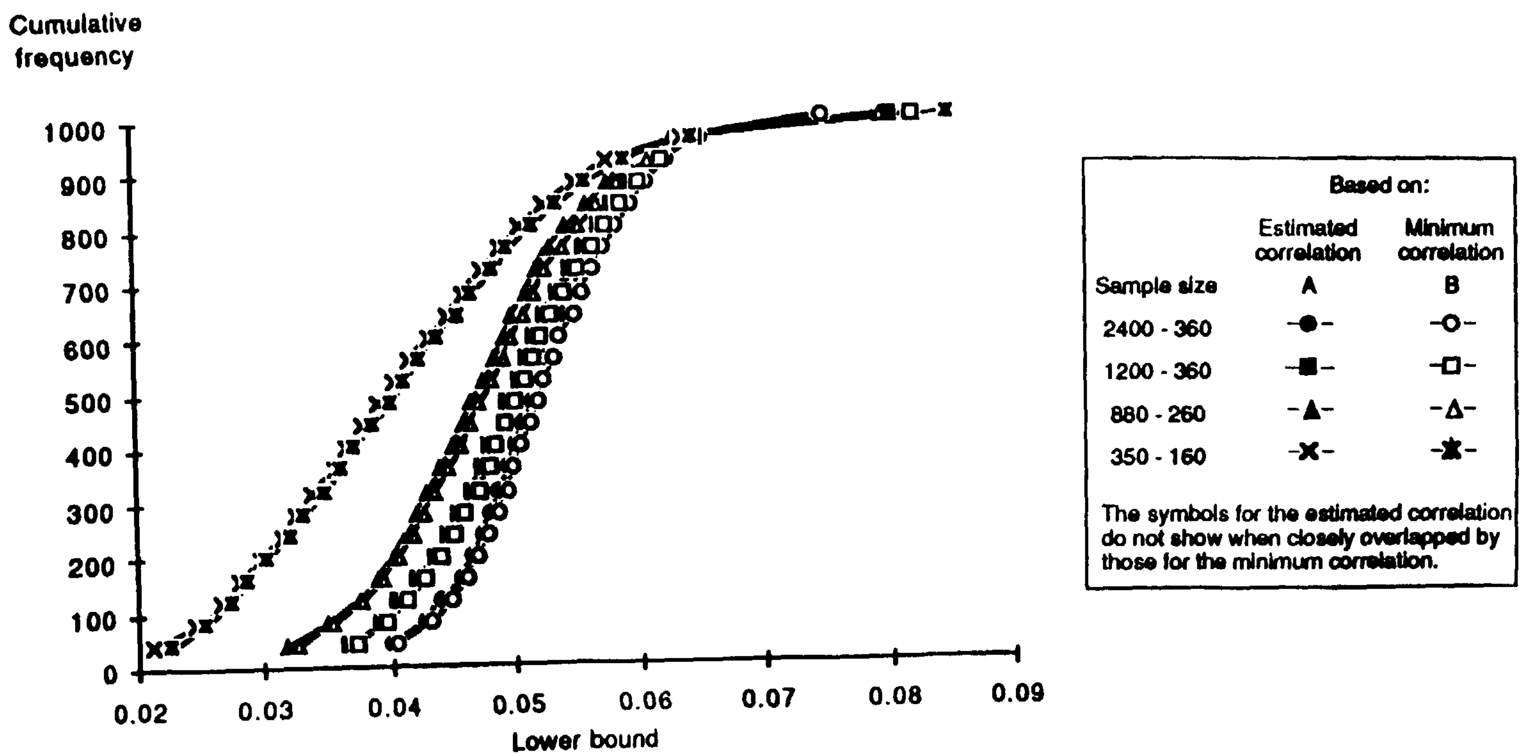
Figure D-3C. Cumulative distribution of the nominal 95 percent lower confidence bounds of the payment error rate using (A) the estimated correlation, and (B) the minimum correlation rule, in the regression estimate of variance, for Population C (based on independent simulations of 1000 samples)



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APPENDIX E

EXPLORATION OF SOME ALTERNATIVE PROCEDURES FOR COMPUTING POOLED VARIANCE ESTIMATES

E.1 Introduction

The current practice in the AFDC quality control program is to estimate the variance of the overpayment error rate for each state using only the data provided by the sample for that state in the current period. It seems likely that the mean square error of the estimated variance could be reduced by somehow making use of additional data. The additional data might be:

(a) Data for the same state for prior periods; or

(b) Data for other (presumably similar) states.

We refer to variance estimation procedures that utilize data from prior time periods or from other states as pooled variance estimation procedures.

Three principal uses for an estimated variance of an estimated overpayment error rate are:

(1) To provide a general measure of precision of the estimated overpayment error rate. Examples of this are to indicate the approximate magnitude of the sampling variability of an estimated overpayment error rate, or to compare the precision of estimates for different states, or to compare the precision of different allocations of the sampling effort to the state sample and to the Federal subsample for a state.

(2) To provide a lower confidence bound for an overpayment error rate. Consideration has been given to the use of a lower confidence bound in various ways in the computation of disallowances, as discussed, for example, in Chapter 3 of this report.

(3) To predict for a future year, the sampling errors that would result from specific sizes of Federal and state samples for a state, or alternatively, to determine for a future year, the approximate sample sizes needed to achieve a specified level of precision.

The pooled variance estimation procedures that we discuss in this appendix will be especially useful for purposes (1) and (3). We have already shown in Sections 2.3 and 2.4 that for purpose (2) the direct estimate of variance based only on data for the current year for a state, presumably (but not necessarily) using a transformed Jackknife variance estimator, is a preferred procedure for computing lower confidence bounds. As discussed in Section 2.5, such a procedure provides a more stable lower confidence bound than would the use of the unknown true variance of the overpayment error rate, even if it were known, or than would result from the use of a pooled variance estimate.

In this appendix, we provide descriptions and approximate evaluations of some alternative procedures for pooled unit variance estimation.

E.2 Variance Estimates Using Data for the Same State for Prior Periods

Alternative (a) mentioned in the introduction to this appendix suggests the possibility of using the regression of the unit variance (defined as the estimated variance of the estimated overpayment error rate multiplied by the Federal subsample size) on other current and recent past data for the same state. We tested this procedure by using the data for the 50 states and the District of Columbia for the four six-month periods in fiscal years 1981 and 1982. The regression was estimated from the data for the first three of the four periods. The regressor (independent) variables were:

- The estimated overpayment error rate for period 3;
- The estimated overpayment error rate for period 2;
- The estimated overpayment error rate for period 1;
- The estimated unit standard error for period 2; and

- The estimated unit standard error for period 1.

The regressand (dependent) variable was the estimated unit variance for period 3. No weights were used in computing the regression. The estimated multiple correlation was .87, indicating that about three-fourths of the variance of the estimated unit variances in period 3 among the states was explained by the regression. It may be seen from the Technical Note at the end of this Appendix that the independent variables for period 1 made trivial contributions to the regression estimates.

Of course, the predictive value of a regression equation appears to be higher for the data used in computing the regression coefficients than will be the case when tested with an independent sample from the same population. An independent sample for the same period is not available. However, a useful test of the effectiveness of the regression procedure is to apply it to data for a succeeding period. Consequently, an estimate of the variance for each state was computed for period 4 by applying the regression coefficients that had been computed for period 3. The regressor variables were now the estimated overpayment rates for periods 4, 3, and 2, and the estimated unit variances for periods 3 and 2. For period 4, the estimated multiple correlation was .68, indicating that about one-half of the variance among the states was explained by the regression. Figure E-1 illustrates, with scatter charts, the relationship of the direct and regression estimates of the unit variances, for states, for both periods 3 and 4. Table E-2 in the Technical Note for Appendix E shows, by states, the values of the dependent and independent variables used in the regression, as well as the unit variances estimated from the regression for periods 3 and 4.

We note that if a predicted value were a perfect prediction of the true unit variance for a state, the correlation between the predicted and the direct variance estimate could not be high if the direct estimates are subject to large variances, as indeed they are. Nevertheless, if a prediction method based on independent data yields a higher correlation with the direct estimates than does a different prediction method, also based on independent data, the higher correlation is evidence of the greater precision of that method. We also note that since this

particular regression approach involved the use of the estimated error rate for the current period as an independent variable, the result is a higher correlation and a higher fraction of variance explained than would be the case if the current error rate were not used as an independent variable. Moreover, since the independent variables used in the regression predictions are subject to large variances, we believe, without further evaluation, that this regression approach for utilizing prior years' data provides a less promising prediction method than the alternatives we discuss below, which employ pooled variance estimates across a considerable number of states.

E.3 Pooled Variance Estimates for Groups of States

Alternative (b) mentioned in the introduction suggests the possibility of using a *composite estimator* of the variance, that is, a weighted mean of the direct estimate for the state and the average of the estimates for some group of states that are similar to the given state in the sense that their average unit variance for recent prior periods was approximately the same. The weights would be chosen so as to minimize, so far as feasible, within each group of states, the mean square error of each estimated state unit variance. To experiment with this idea, the groups were determined by sequencing the states according to the average value of the estimated unit variance in fiscal years 1981 and 1982. Composite variance estimates for fiscal year 1984 were to be made using these groups. We note that we use data for fiscal years 1981 and 1982 to group states for making variance estimates for fiscal year 1984. In practice, the prior years' data might or might not be available for such a grouping. Later, we test the method by examining how well the pooled variance estimates for fiscal year 1984 serve as predictors of the variances for 1983. It would have been desirable to use 1985 data (which were not available). Consequently, 1983 serves as a proxy for 1985.

Figure E-2 shows the average unit variance for the states, arranged according to the value of the average unit variance in 1981 and 1982, as well as the groups that were defined.

On the basis of this graph:

- The first group was defined to consist of the first 10 states;
- The second group consists of the 11th through the 21st state;
- The third group consists of the 22nd through the 31st state;
- The fourth group consists of the 32nd through the 41st state; and
- The fifth group consists of the 42nd through the 51st state.

The states assigned to each of the five groups can be seen by referring to Table E-3, where the states are ordered by group, with a space between groups. For each state, the composite estimate was the weighted mean of the direct estimate of the unit variance for the state and the weighted average of these estimates for the other states in its group, under the condition that the other states had a Federal subsampling rate the same as that of the specified state.

Each group of states was then used to make a pooled unit variance estimate for the current period for each of the included states. The pooled variance estimate for state k within a group is made by taking a weighted average of the current unit variance estimate for the particular state (state k) and the pooled unit estimate for the other states in the group. More specifically, the pooled unit variance estimate for state k is obtained by computing the weighted average

$$\tilde{s}_k^2 = w_k s_k^2 + (1-w_k)s_{ok}^2 ,$$

where s_k^2 is the estimated unit variance of $\hat{R}_k$ (computed as in the present AFDC procedure) for the current period for state k, s_{ok}^2 is the weighted average (weighted by the Federal subsample size) of the unit variance estimates for the current period for the other states in the group (excluding state k). In this computation for state k, the unit variance estimate for each of the other states is modified by replacing its Federal subsampling rate by the rate used for state k.

This pooled estimate will considerably improve the unit variance estimate for state k provided that the true and unknown unit variance in each of the other states in the group is not too different from S_k^2, the true (unknown) unit variance for state k. The improvement results because s_{ok}^2 is estimated from a much larger sample of cases than is s_k^2. Of course, s_{ok}^2 is, in fact, a biased estimate of S_k^2, the bias depending on how much the expected value of s_{ok}^2 differs from S_k^2. The weight w_k for state k can be chosen, as described in the Technical Note to Appendix E, so as approximately to minimize the mean square error of s_k^2 as an estimate of S_k^2, taking account of approximate measures of the bias as well as the variances involved.

We note, especially, as seen in the Technical Note, that in order to compute approximately optimum values of w_k for a state, estimates are needed of the unit variance for each state, as well as of the bias of s_{ok}^2 as an estimate of s_k^2. Of course, we do not know the values of these terms and must estimate them. We have used approximate procedures to do this, as discussed in the Technical Note. In particular, the bias could be estimated directly for each state, but such estimates are subject to variances that are too large to be useful. Consequently, we examine the implications of some alternative procedures for determining an approximately optimum w_k.

As seen in the Technical Note, the estimates of the average squared bias were negative for four of the five groups, and positive for one. While the true squared bias must be zero or positive, negative *estimates* are possible. These estimates, even the average for a group of about 10 states, are still subject to very large sampling errors. Of course, the negative estimates are the result of sampling error, and we regard the positive ones as also substantially determined by sampling variability. Consequently, we have used two different measures of bias that result in two sets of approximately "optimum" weights. For one set, we used an estimate of zero bias for each state. As another alternative, we use for each state a high average

squared bias estimate obtained as the average of the absolute values of the estimated squared biases of the five groups.

The manner in which the weights in the composite estimator were determined, so as approximately to minimize the average mean square error for the states in the group, is detailed in the Technical Note at the end of this appendix.

Tables E-3 and E-4 display, for the alternative estimates of optimum weights, the composite estimate of the unit variance in fiscal year 1984 for each of the 50 states and the District of Columbia. The tables also show for each state, among other things, the size of the Federal subsample (n'), the weight used in the composite estimator, the direct estimate of the unit variance, the variances of the estimated average variance in the group and of the direct variance estimate, and the variance of the composite estimate of the unit variance. The definitions and the estimation procedures are given in the Technical Note.

In addition, as a fourth and simpler alternative pooled variance estimation procedure, we have made pooled estimates of the unit variance of the Federal overpayment errors, s_x^2, of the average payment error, $\bar{t}$, and of the estimated correlation of the Federal and the state determinations of overpayment errors, r. These estimates were pooled over all states in the group. The simple pooled unit variance estimate for a state is then

$$\frac{s_x^2}{\bar{t}^2}\left\{1 - r^2(1 - f_i^2)\right\}$$

where $f_i = n'_i/n_i$ is the subsampling fraction for the Federal subsample in the state. This procedure provides what we refer to as a *simple pooled variance estimate,* and is similar but not equivalent to the assumption of zero bias in the computation of optimal weights. Table E-5 displays the simple pooled estimates of unit variances.

In an effort to evaluate the two alternative composite variance estimators, we have made approximate estimates of their variances. We refer to these estimates of the variance of the estimated variances as "experimental" estimates. This term has been used because we have not made these estimates directly from the sample data. Instead, as discussed in the Technical Note, we have derived them from the assumption that the relvariance of the direct estimate, $s^2_{\hat{R}}$, of the variance of $\hat{R}$, the regression estimator for a state from a double sample, can be approximated by

$$\sigma^2_{s^2_{\hat{R}}} = \frac{\tilde{\beta}-1}{n'} (\sigma^2_{\hat{R}})^2 \quad . \tag{1}$$

The value of $\sigma^2_{\hat{R}}$ is estimated directly from the sample data by $s^2_{\hat{R}}$. Approximate values for $\tilde{\beta}$ are derived from the estimates of the variance of variances that have been obtained from the 1000 replicated samples from each of the three test populations, for various sizes of state samples, n, and of Federal subsamples, n'.

We did not make direct analytic estimates of the variance of the variance of the regression estimator for a double sample because the theory is not available. We did not regard it as worth the effort to develop the theory at this time because we believe our "experimental" estimates provide an acceptable alternative, and perhaps a better alternative than direct estimates which would be subject to very large variances.

The estimated values of $\tilde{\beta}$ are shown in Table C-1 and are also discussed in the Technical Note in Appendix C. A linear regression on the Federal subsampling rate was fitted to these values of $\tilde{\beta}$ and used to compute approximate values of $\tilde{\beta}$ for each state. These are displayed in Tables E-3 and E-4. These and the estimated unit variances were then substituted in Equation (1) above to compute the "experimental" values of the variance of the estimated unit variance for each state. The variances of the composite estimate of unit variances were derived from these, as explained in the appended Technical Note.

We now present two kinds of evaluations of the pooled variance estimators. From Figure E-3 (each point represents the ratio for a state), it is seen that the ratio of the estimated variance of the direct estimate to the estimated variance of the composite estimate with zero as the estimate of bias squared varies from an average of approximately a factor of 14 (varying from about 12 to 17) for states with annual Federal subsamples of 150 to an average of approximately 8 (varying from about 6 to 10) for states with a Federal subsample size of approximately 360. Thus, the variance of the composite estimate using zero as the squared bias is small, very substantially below that of the direct estimate of the variance.

The simple pooled variance estimator yields results that are very close to those for the composite estimator using zero as the squared bias, so the variance reductions for the simple pooled variance estimator are similar to those shown in Figure E-3 for the "zero bias" estimator. In fact, it is shown in the Technical Note that the correlation, across states, of the simple pooled variance estimates with those from the composite estimate using zero bias squared is approximately .98. This correlation is high enough that we regard it as not worthwhile to make a separate evaluation of the variances of the simple pooled variance estimator.

We note that while the reductions in the variance of the variance estimates are substantial for all Federal subsample sizes, they are greatest for the states in which the Federal subsample is relatively small, and in which reductions in the variance of the variance estimates are most needed. We also note that these results are based on the approximate experimental variance of variance estimates, as discussed earlier. However, since these results depend importantly on the sample sizes involved, the ratios displayed in Figure E-3 should be reasonably close to what they would be if the true variances of the variance estimates were known.

Figure E-3 also displays the ratios of the variance of the direct variance estimate to the variance of the composite variance estimate using the high estimate of the squared bias. The resemblance of the simple pooled estimator to the composite estimator using zero squared bias is a consequence of the similarity in the weights assigned to the direct estimate of the unit variance in these two estimators.

In F: :e E-4, we show the weight assigned to each state for each of four estimators of the variance of the estimated unit variance. (The estimator designated "adjusted simple pooling" is described in Section 2.5.1 of this report.) In this figure, e states are arranged in order of the weights assigned in the simple pooling. We ›te that the weig‹ ts are nearly identical for the simple pooled estimator and the composite estimator using zero squared bias. The weights for the composite estimator us ng the "high" squared bias are much greater, and therefore, result in less variance redu tion. Consequently, from the point of ew of variance reduction, there is a considerable advantage in using the zero bias squared in the composite esti ator versus the alternative high bias squared estimator that we have evaluated. The adjusted pooled estimator assigns weights that are slightly less than twice those assigned by the simple pooled estimator.

The next point to uate is how well the direct es nate of the unit variance, and each of the pooled variance estimates, serves as an estimate of the unknown true unit variance for each state. We cannot make this evaluation directly but can do it indirectly. We have shown in the Technical Note that, without knowing the true variances for 1983, we can approximate the correlation, across states, between the true state unit variances for 1983 and the variance estimates for 1984, for each variance estimation procedure.

Table E-1 summarizes the indicated estimated coefficients of correlations (r), and their squares (r^2), called coefficients of determination, obtained as described in the Technical Note. These are estimated unweighted correlations across states – a small state and a large one have equal weights.

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Table E-1. Estimated unweighted correlations of true unit variance of $\hat{R}$ for 1983 with estimated unit variances for 1984

	Estimated coefficient of correlation r	Estimated coefficient of determination r^2
Estimated unweighted correlation of true unit variance of $\hat{R}$ for 1983, with:		
(a) Direct variance estimate for 1984	.64	.41
(b) Composite variance estimate for 1984 using zero squared bias	.69	.47
(c) Composite variance estimate for 1984 using high squared bias	.75	.57
(d) Simple composite variance estimate for 1984	.69	.47

These correlations are reasonably high, although not as high as would be desirable. About half of the unweighted variance between states of the true unit variance is accounted for by each of the three pooled variance estimators, indicated by the squared correlations. The correlations for the pooled variance estimators are somewhat higher than the correlation for the direct variance estimation (although this may result from sampling variability). This fact, together with the fact that their variances are very much smaller, is sufficient to indicate the substantial advantages of using a pooled variance estimator for general precision measures, for predicting needed sample sizes, or for predicting the precision to be obtained from specified sample sizes in a future year.

We note that it would be desirable, also, to estimate the correlations of the 1984 true state unit variances with the various 1984 variance estimators. We are not able to do this because we do not have independent direct variance estimates for 1984. Nevertheless, it is obvious that the correlations of 1984 true unit variances with the 1984 variance estimates would be higher than those shown in Table E-1.

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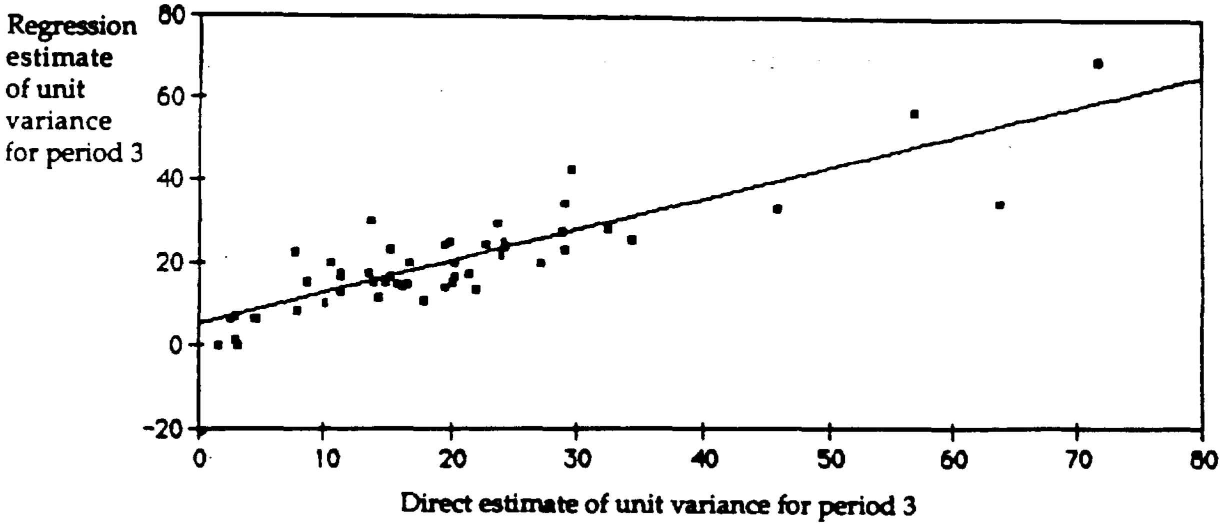
On the evidence presented, it appears that the simple pooled variance estimator might reasonably be regarded as the preferred one among the three estimators we have evaluated. Since this estimator is almost identical to the composite estimator using zero squared bias, the gains in variance reduction will be substantial, as indicated by Figure E-3. Its estimated correlation with the 1983 true values is lower than that of the composite variance estimator with the high squared bias. The gain in correlation with the latter (which may be real or the result of sampling error) seems not to be worth the substantial additional computation complexity involved in computing the composite variance estimates. The simple pooled variance also has the advantage of providing separate estimates of the variance components in the regression estimator (i.e., $s_x^2 \,/\, \bar{t}^2$ and r) for use in evaluating alternate allocations to the state sample and the Federal subsample.

It is possible that, on further analysis, an estimator intermediate between the simple pooled variance estimator and the composite estimator with high bias squared would be found to have additional advantages. We have described such an alternative in Section 2.5, and the weights assigned by such an estimator are shown in Figure E-4. It seems likely that it would have minor advantages over the simple pooled variance estimator as defined and evaluated here. When data for an additional year become available, such a modified simple pooled variance estimator may reasonably be evaluated in comparison with those shown here.

We conclude, then, that for the present, the simple pooled variance estimator (or the modifications of it, as described in Section 2.5.1 of the report) is to be preferred for most variance estimation purposes other than for the computation of lower confidence bounds. The advantages, for these purposes, over the direct variance estimator are substantial

Figure E-1. Scatter charts illustrating the relationship between the direct estimate of variance and the estimate based on the regression, for states, for periods 3 and 4



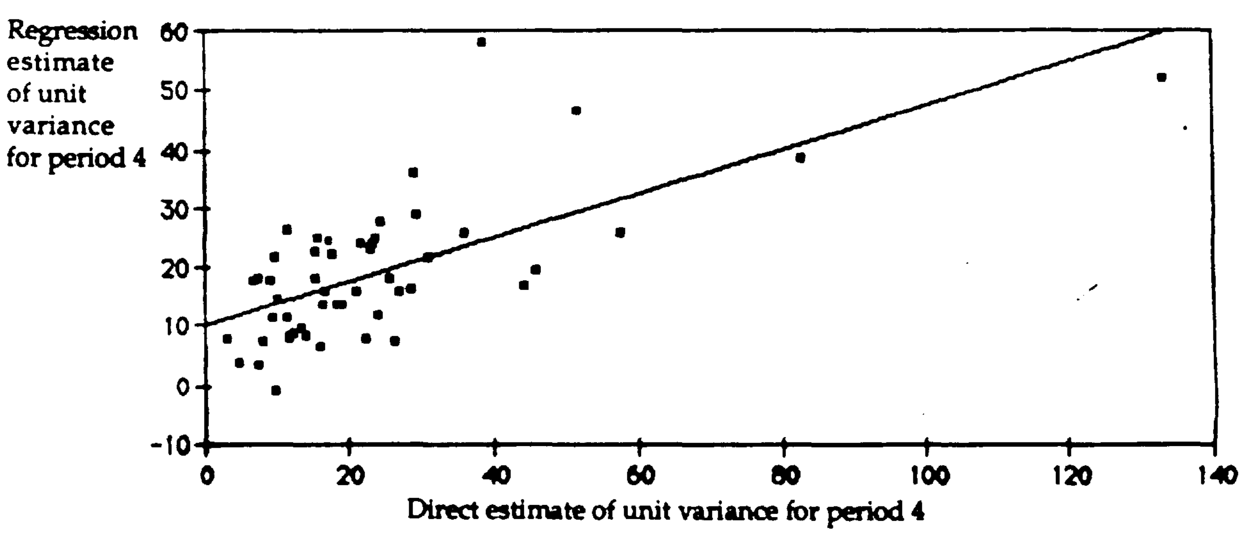


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Figure E-2. Average unit variance in FY 1981-82, for states arranged by that average

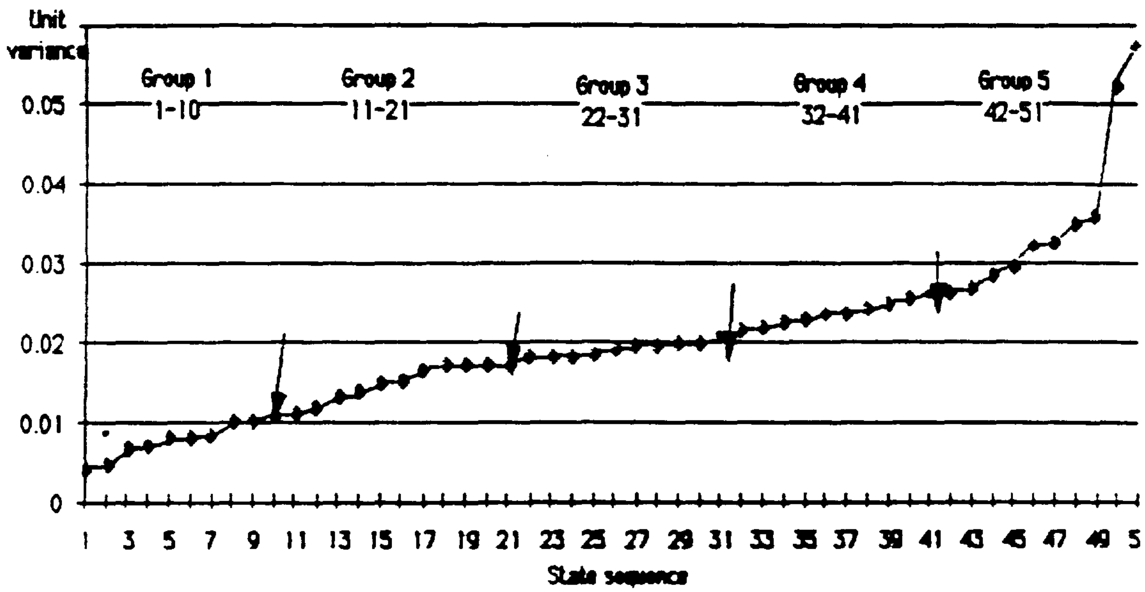


Figure E-3. Ratio of the variance of the direct estimate of unit variance to the composite estimate of unit variance using zero and high squared bias, related to the size of the Federal subsample

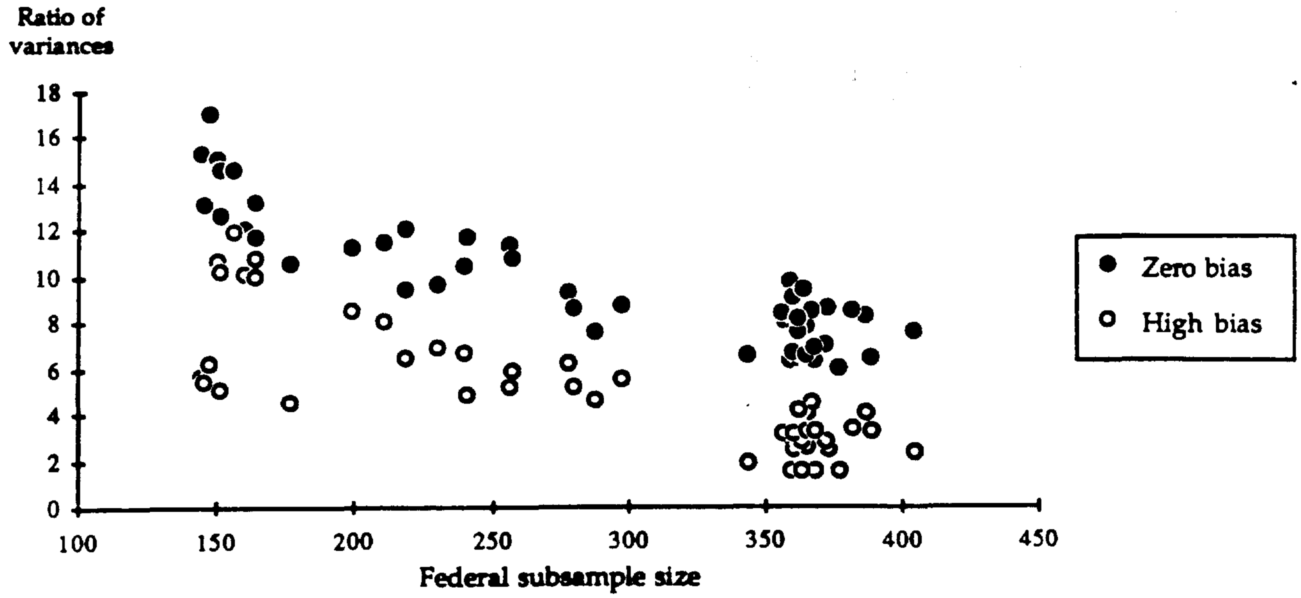
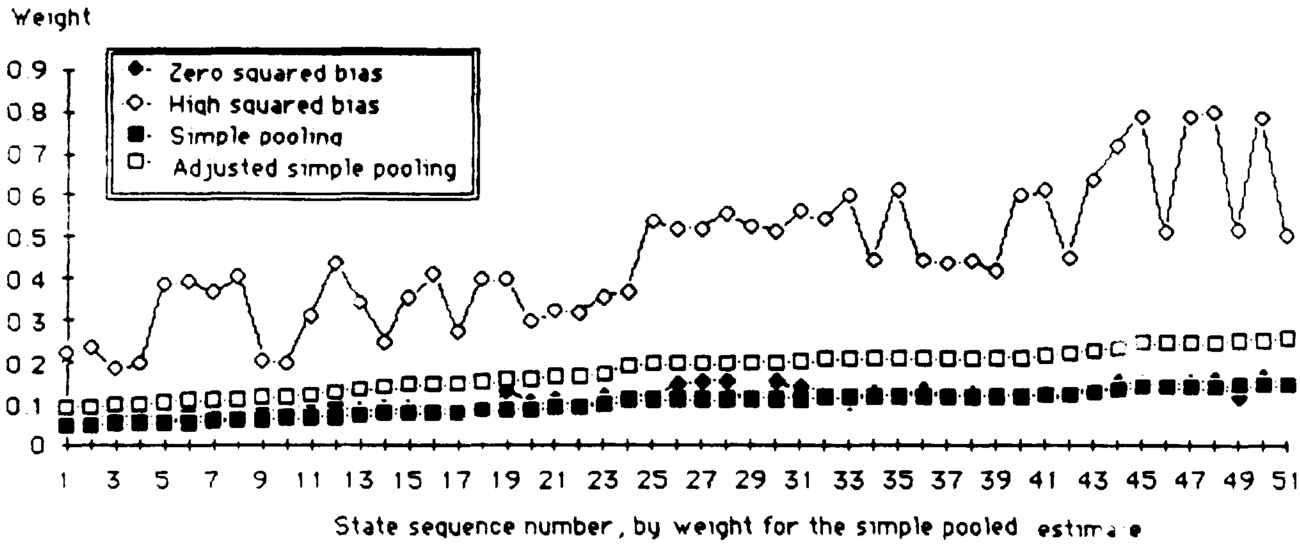


Figure E-4. State weights for pooled unit variance estimates, for states sequenced by weight for the simple pooled estimate



TECHNICAL NOTE FOR APPENDIX E

This note gives details on the computations referred to in Appendix E.

Regression Estimator of the Unit Standard Error of the Payment Error Rate

We are concerned here with the regression of the unit variance (defined as the variance of the estimate of the payment error rate, multiplied by the Federal sample size) on the estimated error rates in the same and the previous two six-month periods and on the unit variances in the previous two periods. In matrix notation, we wish to fit the model

$$y = X\beta + \varepsilon$$

where X is a matrix of 51 rows (the 51 states) and six columns (corresponding to the constant term and the five regressor variables as defined in Appendix E), and y is the column vector of the unit variances. We have estimated the regression coefficient vector β by (unweighted) least squares, namely by

$$b = (X^T X)^{-1} X^T y \, .$$

The computations were made using the data for the first three of the four periods available, yielding the following solution for the vector b:

-0.0005	Constant term
0.2873	Payment error rate, period 3
-0.0090	Payment error rate, period 2
-0.0033	Payment error rate, period 1
0.2941	Unit standard error, period 2
-0.0000	Unit standard error, period 1

The regression estimates of the unit variance in period 3 varied among the 51 states from 0 to 0.069, with a mean value of 0.020 and a standard deviation of 0.013.

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Table E-2 gives the data and the results of the regression value of the unit variance for period 3 as well as the calculated unit variance for period 4 using the coefficients given above. The regression estimate of the unit variance in period 4 varied among the states from 0 to 0.058, with a mean value of 0.019 and a standard deviation of 0.012.

Composite Estimator of the Unit Variance

We consider first the general problem in which a composite estimate $\tilde{x}_i$ for the i-th locality of a group of localities is a weighted mean of a local unbiased estimate x_i and the mean of the estimates x_j of the other localities that are members of the same group. Let m denote the number of localities in the group. The composite estimator for the i-th locality is defined by

$$\tilde{x}_i = W_i x_i + (1-W_i)\,\bar{x}_{(i)} \tag{1}$$

where $\bar{x}_{(i)}$ denotes the mean of the estimates for the m-1 localities other than the i-th locality. We wish to determine the weight W_i that minimizes the mean square error of the composite estimator. We have

$$\mathrm{MSE}(\tilde{x}_i) = \mathrm{Var}(\tilde{x}_i) + (E\tilde{x}_i - Ex_i)^2 \tag{2}$$

$$= W_i^2\sigma_{x_i}^2 + (1-W_i)^2\sigma_{\bar{x}_{(i)}}^2 + (1-W_i)^2\,(E\bar{x}_{(i)} - Ex_i)^2 \,.$$

The value of W_i that minimizes the mean square error is obtained by equating to zero the derivative of the mean square error with respect to W_i:

$$0 = 2W_i\sigma_{x_i}^2 - 2(1-W_i)\,\{\sigma_{\bar{x}_{(i)}}^2 + (E\bar{x}_{(i)} - Ex_i)^2\} \,.$$

Solving this equation for W_i yields the optimum value

$$W_i = \frac{\sigma^2_{\bar{x}_{(i)}} + \left(E\bar{x}_{(i)} - Ex_i\right)^2}{\sigma^2_{x_i} + \sigma^2_{\bar{x}_{(i)}} + \left(E\bar{x}_{(i)} - Ex_i\right)^2} . \tag{3}$$

The parameters in the equation for the optimum $\tilde{W}_i$ are not known, so that estimates of them are used to obtain an estimate of the optimum weight.

In our case, x_i is the estimated unit variance s_i^2 of the estimated payment error rate $\hat{R}_i$ for state i. We make the assumption

$$\sigma^2_{s_i^2} = (\beta_i - 1)\, \sigma_i^4 \,/\, n'_i \tag{4}$$

where β_i is a specified constant for each state i and σ_i^2 is the unit variance that is estimated by s_i^2. This relationship would hold for simple random sampling with replacement.[1] For the regression estimator with double sampling, as used in AFDC, it is an approximate relationship. The specified β_i for each state are shown in Tables E-3 and E-4. The values of β_i were computed from the observed relationship of β_i (as given by the approximation $\beta_i = 1 + n'_i s^2_{x_i}/s^2_{\hat{R}_i}$), that is yielded by Equation (4), to the Federal subsampling rates n'/n in the Test Populations A, B, and C. A linear regression equation was fitted to the data shown in Table C-1 in Appendix C. The dependent variable was the $\tilde{\beta}_i$ shown in the table, and the independent variable was $f_i = n'_i/n_i$. The resulting regression equation was

$$\beta_i = 64.3 - 54.47 f_i .$$

[1] Hansen, M.H., Hurwitz, W.N., and Madow, W.G., ***Sample Survey Methods and Theory***, Vol. I, p. 427 (New York: John Wiley & Sons, 1953)

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We then define the estimator

$$s^2_{x_i} = (\beta_i - 1)\,\{u_i(1 - (1 - n'_i/n_i)r^2_i)\}^2/n'_i \qquad (5)$$

where u_i denotes the ratio of the estimated variance of the Federal determination of the overpayment errors to the square of the average payment error estimated from the state sample, and r_i denotes the estimated correlation between the Federal and state determinations of the overpayment errors. The expression in the braces divided by n'_i is the appropriate regression variance estimator of the payment error rate, $\hat{R}_i$, as used by AFDC.

Groups of states were defined in the following way. For each state i, for each six-month period t in fiscal year 1981-82, the unit variance was computed as

$$s^2_{ti} = u_{ti}\,(1 - .8r^2_{ti})$$

where the u_{ti} and r_{ti} are defined as u_i and r_i in Equation (5). This computation of the unit variance replaces the Federal subsampling rate that was used for the state by the constant rate .2, which is roughly the average Federal subsampling rate. The average unit variance for state i in fiscal year 1981-82 was then taken as the weighted mean of the four six-month periods, viz.,

$$s^2_i = \sum_{t=1}^{4} n'_{ti}\, s^2_{ti} \Big/ \sum_{t=1}^{4} n'_{ti}$$

where n'_{ti} denotes the Federal sample size in period t. The states were ordered by the value of s^2_i and five groups were defined as exhibited in Figure E–2.

For the set of states in a group *other* than the state i, the average variance is

$$\bar{x}_{(i)} = \sum_{j \neq i} n'_j u_j\,[1 - (1 - n'_i/n_i)r^2_j]/(n' - n'_i) \qquad (6)$$

whose variance is estimated by

$$s^2_{\bar{x}_{(i)}} = \sum_{j \neq i} n'_j(\beta_j-1)\{u_j[1-(1-n'_i/n_i)r^2_j]\}^2/(n'-n'_i)^2 . \tag{7}$$

The term $\{E\bar{x}_{(i)}-Ex_i\}^2$ in Equation (3) is the square of the bias that results when the average variance for the *other* states in a group is used as an estimate of the variance for state i in the group.

To estimate $(E\bar{x}_{(i)}-Ex_i)^2$, we note that

$$E(\bar{x}_{(i)}-x_i)^2 = E\{(\bar{x}_{(i)}-E\bar{x}_{(i)})-(x_i-Ex_i)+(E\bar{x}_{(i)}-Ex_i)\}^2$$

$$= \sigma^2_{\bar{x}_{(i)}} + \sigma^2_{x_i} + (E\bar{x}_{(i)}-Ex_i)^2$$

since x_i and $\bar{x}_{(i)}$ are independent. An unbiased estimate of the desired parameter, termed the square of the bias, is then given by

$$(\bar{x}_{(i)}-x_i)^2 - s^2_{\bar{x}_{(i)}} - s^2_{x_i} .$$

This could be computed directly for each state, but such estimates are subject to extremely large variances, too large to be useful. Instead, we consider, as a first alternative, using for each state the average squared bias for the whole group of states. We would therefore estimate this parameter for a group by

$$b^2 = \Sigma_i n'_i\{(x_i-\bar{x}_{(i)})^2 - s^2_{x_i} - s^2_{\bar{x}_{(i)}}\} . \tag{8}$$

Although the parameter being estimated is non-negative, the estimate b^2 may take on a negative value for a group. In such a case, b^2 may be taken to have the value zero for the group. As a second alternative, because even the group averages are subject to wide sampling variation, the values of b^2 may be taken to be the average over all the groups. Even the average may be negative, in which case we may take $b^2=0$. Substituting the estimates of the parameters in Equation (3), we obtain the estimates of $\tilde{W}_i$.

A further modification is suggested by the fact that the first term in the denominator of W_i, namely $\sigma^2_{x_i}$, is subject to a quite large variance. We therefore consider replacing the estimate $s^2_{x_i}$ by a more stable estimate of $\sigma^2_{x_i}$ in the following way. We first replace the quantity within the braces in Equation (5) by the average of such quantities for the other states in the same group; the latter is given by $\bar{x}_{(i)}$ of Equation (6). We then define the more stable estimator as the weighted mean of the new variance computed by Equation (5) and the direct estimate of variance for the state. Thus, we have

$$\hat{s}^2_{x_i} = \frac{(n'-n'_i)\ [(\beta_i-1)\ /\ n'_i]\ \bar{x}_{(i)} + n'_i\, s^2_{x_i}}{n'} \ .$$

This is then substituted for $\sigma^2_{x_i}$ in Equation (3).

The various parameters as discussed above were estimated for each state from the state data for fiscal year 1984, based on the groups of states as defined above and displayed in Figure E-2 and Tables E-3 and E-4. The average value of b^2 turned out to be negative. Table E-3 gives the composite estimates when b^2 is taken to be zero for each group. Table E-4 gives the composite estimates when b^2 is taken to be the weighted mean of the absolute values of the value of b^2 computed for each group. We refer to this as the "high" squared bias, because it is likely to be greater than the true squared bias (since its expected value is greater). In addition to the composite estimate of the unit variance for each state, Tables E-3 and E-4 display the

size of the Federal subsample, n', the values of β (beta), the estimated error rate, the weight used in the composite estimator, and the experimental estimate (described below) of the variance of the estimated unit variances for both the direct estimate and the composite estimate.

The weight calculated for a state is considerably greater when the "high" squared bias is used than when a zero squared bias is used. The true optimum weight is somewhere between the two, since the true squared bias is undoubtedly positive. Figure E-5 is a scatter diagram which shows the relationship of the weights for zero and high squared bias. We note that, on the average, the weight is about four times higher when the high squared bias is used. Figure E-4 also shows this relationship.

Because the composite estimator involves considerable computation, we consider also a simple pooled estimate of the unit variance. Groups of states are defined as above for the composite estimator. For state i of group g, the simple pooled estimator of the unit variance is given by

$$\frac{s^2_{gx}}{\bar{t}^2_g} \left\{ 1 - \left(1 - \frac{n'_{gi}}{n_{gi}} \right) r^2_g \right\} . \qquad (9)$$

In this expression, n_{gi} and n'_{gi} denote the sizes of the state sample and the Federal subsample, respectively. The other quantities are weighted means of corresponding quantities for all states in the group. Specifically,

$$s^2_{gx} = \sum_i^{m_g} (n'_{gi} - 1)\, s^2_{gix} \,/\, (n'_g - m_g)$$

$$s^2_{gy} = \sum_i^{m_g} (n'_{gi} - 1)\, s^2_{giy} \,/\, (n'_g - m_g)$$

$$s_{gxy} = \sum_i^{m_g} (n'_{gi} - 1)\, sgixy \,/\, (n'_g - m_g)$$

r_g = $s_{gxy} / s_{gx} s_{gy}$

m_g = number of states in group g

n'_g = $\sum_{i}^{m_g} n'_{gi}$

s^2_{gix} = estimated unit variance of the Federal determination of payment error

s^2_{giy} = estimated unit variance of the state determination of payment error, as estimated from the Federal subsample

s_{gixy} = estimated unit covariance of the Federal and state determinations of payment error.

Table E-5 displays the simple pooled estimates for each state. These closely resemble the composite estimates using zero squared bias, as exhibited in Figure E-6. The correlation between the two state estimates is .978. On the average, the simple pooled estimate is about 10 percent greater than the composite estimate.

The variance of an estimate of the unit variance for a state is a function of the size of the sample used to estimate the unit variance. In Figure E-3 we show, by state, the ratio of the direct estimate for fiscal year 1984 to the composite estimate (using zero squared bias and the high squared bias) as related to the Federal sample size. The relationship, as expected, appears to be a monotone decreasing function of the sample size, concave upward, and somewhat flatter when using the high squared bias.

An important reason for seeking a better estimate of the true unit variance in a given year is to predict the unit variance in a subsequent year, for the purpose of determining the sample sizes that will yield estimates of the payment error rate of some prescribed precision. In the discussion above, we have used data for fiscal years 1981 and 1982 to group states, and have then estimated unit variance

for 1984. In practice, we would estimate the unit variance for 1983 and use it to predict the unit variance for 1984. Since this should be similar to "predicting" 1983 from the 1984 estimates, we present such analyses here. Figure E-7 presents scatter diagrams showing the relationships of the several 1984 estimates of unit variance to the direct estimate for 1983. As shown in Figure E-7, each of the estimates for 1984 shows a moderate correlation with 1983, of about .5 (ranging from .44 to .52).

To evaluate the 1984 pooled variance estimator as a predictor of the 1983 variance, let

x_{ti} = direct estimate of unit variance for state i in year t, where t=3 for fiscal year 1983 and t=4 for fiscal year 1984;

z_{ti} = pooled estimate of unit variance;

X_{ti} = true unit variance; and

Z_{ti} = expected value of z_{ti}.

We are interested in the correlation, over states, between the direct estimate for 1984 and the true unit variance for 1983, and the correlation between each of the pooled unit variance estimates for 1984 and the true unit variance for 1983. We denote these correlation coefficients by $\rho_{x_4X_3}$ and $\rho_{z_4X_3}$, respectively. We define

Δx_{4i} = $x_{4i} - X_{4i}$

$\overline{X}_4$ = average of X_{4i} across states

$\overline{X}_3$ = average of X_{3i} across states.

The covariance of x_{4i} and X_{3i} across states is defined by

$$\begin{aligned}
\sigma_{x_4 x_3} &= E\,E\{(x_{4i} - \overline{X}_4)(X_{3i} - \overline{X}_3) \mid i\} \\
&= E\,E\{(X_{4i} + \Delta x_{4i} - \overline{X}_4)(X_{3i} - \overline{X}_3) \mid i\} \\
&= E\,E\{(X_{4i} - \overline{X}_4)(X_{3\,i} - \overline{X}_3) \mid i\} \\
&= E(X_{4i} - \overline{X}_4)(X_{3i} - \overline{X}_3) \\
&= \sigma_{X_4 X_3}\ .
\end{aligned}$$

The variance of x_{4i} across states is defined by

$$\begin{aligned}
\sigma^2_{x_4} &= E\,E\{(x_{4i} - \overline{X}_4)^2 \mid i\} \\
&= E\,E\{(X_{4i} + \Delta x_{4i} - \overline{X}_4)^2 \mid i\} \\
&= E\,E\{(X_{4i} - \overline{X}_4)^2\} + E\,E\{(\Delta x_{4i})^2 \mid i\} \\
&= E(X_{4i} - \overline{X}_4)^2 + E\{\sigma^2_{\Delta x_{4i}} \mid i\} \\
&= \sigma^2_{X_4} + \sigma^2_{\Delta x_4}\ \text{, say.}
\end{aligned}$$

Since

$$E(X_{3i} - \overline{X}_3)^2 = \sigma^2_{X_3}$$

we have

$$\rho_{x_4 X_3} = \frac{\sigma_{x_4 X_3}}{\sigma_{x_4} \sigma_{X_3}}$$

$$= \frac{\sigma_{X_4 X_3}}{\sigma_{X_3} \sqrt{\sigma^2_{X_4} + \sigma^2_{\Delta x_4}}}$$

$$= \frac{\sigma_{X_4 X_3}}{\sigma_{X_3} \sigma_{X_4} \sqrt{1 + \sigma^2_{\Delta X_4} / \sigma^2_{X_4}}}$$

$$= \rho_{X_4 X_3} \frac{1}{\sqrt{1 + \sigma^2_{\Delta X_4} / \sigma^2_{X_4}}}. \tag{10}$$

Similarly, it can be shown that

$$\rho_{z_4 X_3} = \rho_{Z_4 X_3} \frac{1}{\sqrt{1 + \sigma^2_{\Delta z_4} / \sigma^2_{z_4}}}. \tag{11}$$

None of the correlations between the values X_3, X_4 and Z_4 can be estimated directly from the data. We can, however, estimate the correlations of their estimates, and similar algebraic manipulation shows that

$$\rho^2_{x_3 x_4} = \rho^2_{X_4 X_3} \frac{1}{\left(1 + \frac{\sigma^2_{\Delta x_3}}{\sigma^2_{X_3}}\right)\left(1 + \frac{\sigma^2_{\Delta x_4}}{\sigma^2_{X_4}}\right)}. \tag{12}$$

$$\rho^2_{x_3 z_4} = \rho^2_{X_3 Z_4} \frac{1}{\left(1 + \frac{\sigma^2_{\Delta x_3}}{\sigma^2_{X_3}}\right)\left(1 + \frac{\sigma^2_{\Delta z_4}}{\sigma^2_{Z_4}}\right)} \quad . \tag{13}$$

Solving Equation (12) for $\rho_{X_4 X_3}$ and substituting into Equation (10), we obtain

$$\rho_{X_4 X_3} = \sqrt{1 + \frac{\sigma^2_{\Delta x_3}}{\sigma^2_{X_3}}} \; \rho_{x_3 x_4} \quad . \tag{14}$$

Similarly, solving Equation (13) for $\rho_{X_3 Z_4}$ and substituting into Equation (11), we obtain

$$\rho_{Z_4 X_3} = \sqrt{1 + \frac{\sigma^2_{\Delta x_3}}{\sigma^2_{X_3}}} \; \rho_{x_3 z_4} \quad . \tag{15}$$

It is necessary to estimate the quantities in these equations. We have

$$E \frac{1}{51} \sum_i^{51} (x_{4i} - \bar{x}_4)^2$$

$$= \frac{1}{51} \sum_i^{51} E\{(x_{4i} - X_{4i}) + (X_{4i} - \bar{X}_4) + (\bar{X}_4 - \bar{x}_4)\}^2$$

$$= \frac{1}{51} \sum_i^{51} \{E(x_{4i} - X_{4i})^2 + (X_{4i} - \bar{X}_4)^2 + E(\bar{x}_4 - \bar{X}_4)^2$$

$$- 2E(\bar{x}_4 - \bar{X}_4)(x_{4i} - X_{4i})\}$$

$$= \frac{1}{51} \sum_{i}^{51} \{ \sigma^2_{X_{4i}} + (X_{4i} - \overline{X}_4)^2 - E(\overline{x}_4 - \overline{X}_4)^2 \}$$

$$= \sigma^2_{\Delta x_4} + \sigma^2_{X_4} - E(\overline{x}_4 - \overline{X}_4)^2 .$$

We ignore the third term of the right member since it is small compared to the first term. Similarly, we have

$$E \frac{1}{51} \sum_{i}^{51} (z_{4i} - \overline{z}_4)^2 \doteq \sigma^2_{\Delta z_4} + \sigma^2_{Z_4} .$$

From Table E-3, we compute the estimates of the quantities involved:

$$\frac{1}{51} \sum^{51} (x_{4i} - \overline{x}_4)^2 = 1.2228 \times 10^{-4}$$

$$s^2_{\Delta x_4} = 6.4484 \times 10^{-5}$$

so that

$$s^2_{x_4} = 5.7796 \times 10^{-5}$$

and

$$\frac{s^2_{\Delta x_4}}{s^2_{x_4}} = 1.1157.$$

We assume that this ratio has the same value for 1983 as for 1984, so that we take

$$\frac{s^2_{\Delta x_3}}{s^2_{x_3}} = 1.1157.$$

From the data we have also estimated

$\hat{\rho}_{x_3x_4}$ = .439

$\hat{\rho}_{x_3z_4}$ = .473 for the composite estimate using zero squared bias

= .519 for the composite estimate using high squared bias

= .473 for the simple pooled estimate.

Substituting the estimates into Equations (14) and (15) yields

	$\hat{\rho}_{z_4x_3}$		
$\hat{\rho}_{x_4x_3}$	Composite zero bias	Composite high bias	Simple pooled
.639	.688	.755	.688

Thus, the composite estimate using zero squared bias and the simple pooled variance estimates for 1984 have the same estimated correlation with the true unit variance for 1983. The estimated correlation with the direct estimate is somewhat lower. It is somewhat higher with the composite estimate with high squared bias. The differences may be real or due to sampling variability. These correlations are about 50 percent greater than the correlation between any of these estimates for 1984 and the direct estimate for 1983.

We return to explain the computation of the variances of the composite estimators, as shown in the last column of Tables E-3 and E-4. These values, which we have termed "experimental," are based on the following speculation. For economy of notation, let s_i^2 denote the variance defined by Equation (5) and $s_{(i)}^2$ the variance defined by Equation (7). Let $\tilde{s}_i^2$ denote the composite estimate of the unit variance for state i, i.e.,

$$\tilde{s}_i^2 = W_i s_i^2 + (1-W_i)\, s_{(i)}^2 \; .$$

Conditional on the value of W_i,

$$\mathrm{Var}(\tilde{s}_i^2) = W_i^2 \ \mathrm{Var}(s_i^2) + (1-W_i)^2\mathrm{Var}(s_{(i)}^2) \tag{16}$$

since s_i^2 and $s_{(i)}^2$ are independent. We take

$$\mathrm{Var}(s_i^2) \simeq (\beta_i-1)\,(s_i^2)^2/n'_i$$

and

$$\mathrm{Var}(s_{(i)}^2) = \sum_{j\neq i} (n'_j)^2\mathrm{Var}(s_j^2) \,/\, (n' - n'_i)^2 .$$

The experimental estimate of the variance or mean square error is given by substituting estimates of the quantities in Equation (16).

The problem with direct variance estimates by states is their greater sampling variability, as discussed in Section 2.5 of the report and in Appendix C. We conclude that the sampling variability of the composite estimator is considerably less, as illustrated in Figure E-3. Consequently, for making estimates of needed sample sizes, at least, the composite estimates are likely to have substantial advantage over the use of the direct state variance estimates.

With the squared biases assumed equal to zero, use of the pooled unit variance estimate for each state results in a mean square error of the variance estimates that varies from about one-sixth to one-fourteenth as large, depending on the size of the state and Federal samples, as the variance of the unit variance estimate based only on the current data for a state. This may modestly overstate the gains. The mean square errors for the estimates assuming biases show mean square error reductions of about half this amount, but these substantially understate the gains because the biases, by design, are substantial overestimates. Clearly, the improvement through pooled variance estimation is substantial for all states, but is of course greatest for the states with the smaller AFDC-QC samples.

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Table E-2. Data and results of regression estimates of variance, by states

	Estimated payment error rate				Estimated unit variance				Regression estimates	
	Period				Period				Period	
State	1	2	3	4	1	2	3	4	3	4
AK	.1376	.2211	.1288	.1104	.04417	.13373	.07181	.05147	.06947	.04653
AL	.0832	.0709	.0551	.0508	.03238	.01607	.01482	.01842	.01522	.01380
AR	.0657	.0701	.0884	.0521	.01705	.00986	.02907	.01493	.02303	.01807
AZ	.0874	.0784	.1155	.1165	.02107	.02190	.02900	.02887	.03421	.03628
CA	.0861	.0500	.0736	.0463	.04103	.01264	.02717	.01636	.01971	.01604
CO	.0998	.0652	.0500	.0800	.04197	.01983	.01643	.01496	.01486	.02273
CT	.0798	.0690	.0528	.0748	.00155	.00999	.01124	.04580	.01280	.01967
DC	.1511	.1198	.1759	.1666	.00896	.04274	.05690	.03832	.05711	.05820
DE	.1285	.1024	.1008	.1357	.06263	.03811	.06372	.13296	.03440	.05206
FL	.0749	.0836	.0631	.0576	.00617	.01429	.01126	.00972	.01691	.01459
GA	.0732	.0577	.0477	.0549	.01105	.00737	.01773	.02082	.01069	.01594
HI	.1012	.1008	.0872	.0770	.03756	.02881	.03234	.05748	.02786	.02609
IA	.0440	.0411	.0406	.0490	.00890	.00500	.00775	.01111	.00820	.01143
ID	.1265	.0507	.0473	.0613	.07180	.02711	.01128	.02674	.01627	.01591
IL	.0860	.0793	.0767	.0883	.02616	.01034	.02030	.03559	.01966	.02596
IN	.0520	.0323	.0345	.0425	.01478	.00511	.00429	.01227	.00653	.00863
KS	.0751	.0870	.0562	.0008	.00967	.03703	.02391	.00004	.02158	.02480
KY	.0550	.0443	.0337	.0378	.00773	.00596	.00443	.00775	.00643	.00729
LA	.0577	.0763	.0645	.0604	.01396	.01300	.02134	.00727	.01705	.01838
MA	.1112	.0735	.0545	.0944	.03411	.01689	.00842	.01699	.01517	.02444
MD	.1179	.1132	.0911	.0733	.01047	.02996	.02363	.01769	.02916	.02239
ME	.0861	.0716	.0526	.0291	.02243	.01710	.01581	.00280	.01479	.00788
MI	.0691	.0767	.0898	.0814	.01040	.03191	.01360	.00946	.02984	.02190
MN	.0381	.0499	.0309	.0297	.01933	.02657	.01415	.02225	.01169	.00783
MO	.0648	.0770	.0611	.0343	.01834	.01609	.01344	.01141	.01695	.00858
MS	.0733	.0649	.0500	.0446	.01431	.02044	.01391	.02405	.01513	.01182
MT	.0688	.0305	.0113	.0384	.02961	.00550	.00303	.02612	-.00006	.00730
NC	.0619	.0465	.0372	.0283	.00859	.00406	.00288	.00452	.00684	.00407
ND	.0330	.0287	.0128	.0254	.01668	.00666	.00284	.00736	.00084	.00350
NE	.0410	.0676	.0586	.1325	.01849	.04252	.01945	.08227	.02417	.03862
NH	.0549	.0771	.0584	.0587	.04010	.00648	.02194	.02564	.01338	.01811
NJ	.0836	.0770	.0936	.0522	.02154	.02009	.02882	.00900	.02741	.01795
NM	.1241	.1236	.1189	.0915	.04409	.04972	.02956	.02926	.04284	.02908
NV	.0250	.0203	.0147	.0104	.01310	.00019	.00152	.00941	-.00041	-.00119
NY	.0912	.0694	.0681	.0913	.01118	.01816	.01055	.02338	.01956	.02407
OH	.0838	.0933	.0769	.0753	.02562	.02783	.01982	.03070	.02474	.02204
OK	.0492	.0829	.0465	.0286	.03647	.03942	.01665	.01143	.01962	.00800
OR	.0670	.0685	.0734	.0679	.01669	.05963	.04594	.02411	.03336	.02771
PA	.0979	.0830	.0937	.0762	.01062	.01364	.03423	.01128	.02544	.02642
RI	.0676	.0573	.0584	.0548	.02607	.01144	.02007	.02837	.01498	.01651
SC	.0739	.0828	.0937	.0839	.01571	.00972	.02264	.01540	.02437	.02522
SD	.0721	.0208	.0376	.0365	.06411	.00378	.01002	.01380	.01001	.00860
TN	.1019	.0771	.0557	.0427	.01251	.02053	.01523	.00928	.01659	.01157
TX	.0711	.0791	.0881	.0790	.02880	.01595	.02411	.02165	.02463	.02431
UT	.0598	.0371	.0543	.0457	.03545	.01057	.01957	.01897	.01375	.01385
VA	.0369	.0349	.0330	.0481	.00867	.00470	.00238	.01301	.00600	.00968
VT	.0382	.0646	.0566	.0327	.01421	.03737	.00749	.01562	.02212	.00645
WA	.0985	.0868	.0731	.0560	.07344	.02723	.02435	.00640	.02348	.01788
WI	.0942	.0771	.0489	.0489	.02155	.01907	.01607	.01607	.01423	.01366
WV	.1894	.0762	.0811	.0838	.10835	.01851	.01519	.02310	.02302	.02314
WY	.0709	.0836	.0385	.0563	.03275	.03759	.02020	.04434	.01671	.01707

Table E-3. Composite estimates of unit variance, using zero squared bias, by states

State	n'	beta	f	Weight	Unit variance: Direct	Unit variance: Composite	Variance of: Group average	Variance of: Local variance	Group average variance	Variance of composite
ND	144	39	.456	.066	.0268	.0146	4.028E-06	5.729E-05	.0137	3.764E-06
NV	151	39	.462	.079	.0146	.0146	4.599E-06	5.336E-05	.0146	4.235E-06
NC	368	56	.147	.158	.0070	.0074	1.550E-06	8.288E-06	.0075	1.306E-06
IA	344	51	.239	.151	.0077	.0095	2.368E-06	1.327E-05	.0098	2.009E-06
VT	145	38	.482	.076	.0185	.0151	4.767E-06	5.770E-05	.0148	4.403E-06
KY	360	56	.156	.156	.0060	.0076	1.666E-06	8.992E-06	.0079	1.406E-06
VA	364	55	.162	.155	.0075	.0078	1.643E-06	8.978E-06	.0078	1.389E-06
IN	377	56	.161	.165	.0034	.0077	1.833E-06	9.258E-06	.0085	1.530E-06
NH	147	39	.473	.059	.0335	.0148	3.889E-06	6.214E-05	.0137	3.660E-06
UT	177	37	.500	.095	.0191	.0155	5.096E-06	4.860E-05	.0152	4.612E-06
MT	150	38	.479	.066	.0350	.0244	1.035E-05	1.455E-04	.0236	9.659E-06
ME	219	46	.335	.083	.0197	.0189	6.669E-06	7.358E-05	.0189	6.115E-06
FL	360	56	.153	.102	.0167	.0122	2.658E-06	2.337E-05	.0117	2.386E-06
AR	241	51	.252	.085	.0113	.0158	4.899E-06	5.258E-05	.0162	4.481E-06
KS	257	48	.298	.087	.0243	.0176	5.460E-06	5.705E-05	.0170	4.983E-06
SD	151	39	.456	.069	.0124	.0231	1.021E-05	1.384E-04	.0239	9.512E-06
LA	373	56	.154	.115	.0137	.0123	2.897E-06	2.232E-05	.0121	2.564E-06
GA	361	56	.146	.110	.0140	.0120	2.729E-06	2.210E-05	.0118	2.429E-06
CT	358	53	.211	.125	.0074	.0143	4.400E-06	3.092E-05	.0152	3.852E-06
MO	405	56	.149	.131	.0112	.0121	3.008E-06	1.999E-05	.0123	2.615E-06
TN	366	56	.159	.120	.0095	.0125	3.236E-06	2.362E-05	.0129	2.846E-06
RI	219	44	.369	.106	.0172	.0217	1.115E-05	9.444E-05	.0222	9.975E-06
SC	363	54	.194	.154	.0099	.0153	6.529E-06	3.587E-05	.0162	5.524E-06
NY	357	56	.148	.118	.0239	.0140	4.251E-06	3.163E-05	.0127	3.747E-06
CO	288	48	.299	.130	.0091	.0189	9.381E-06	6.268E-05	.0203	8.160E-06
MI	364	56	.150	.151	.0129	.0139	5.219E-06	2.945E-05	.0141	4.433E-06
PA	365	56	.148	.106	.0273	.0138	3.830E-06	3.237E-05	.0122	3.425E-06
WI	372	56	.149	.143	.0182	.0140	4.814E-06	2.892E-05	.0134	4.127E-06
AZ	258	49	.286	.092	.0359	.0192	7.216E-06	7.094E-05	.0175	6.549E-06
MS	361	55	.176	.149	.0036	.0144	6.229E-06	3.555E-05	.0163	5.300E-06
MN	366	54	.192	.152	.0038	.0149	6.673E-06	3.713E-05	.0169	5.657E-06
MA	366	56	.149	.127	.0184	.0181	7.176E-06	4.924E-05	.0181	6.263E-06
NJ	362	56	.149	.130	.0148	.0180	7.468E-06	4.994E-05	.0185	6.497E-06
AL	367	55	.179	.116	.0255	.0191	7.184E-06	5.452E-05	.0182	6.348E-06
WV	298	51	.239	.115	.0126	.0209	9.814E-06	7.573E-05	.0220	8.688E-06
OK	278	50	.268	.107	.0067	.0217	1.075E-05	8.958E-05	.0235	9.596E-06
ID	156	37	.495	.069	.0540	.0300	1.545E-05	2.091E-04	.0283	1.439E-05
MD	363	56	.150	.132	.0130	.0181	7.651E-06	5.043E-05	.0188	6.643E-06
WY	164	39	.471	.076	.0334	.0289	1.574E-05	1.922E-04	.0285	1.455E-05
CA	387	56	.151	.120	.0245	.0181	6.534E-06	4.773E-05	.0173	5.747E-06
TX	363	56	.149	.122	.0208	.0181	6.913E-06	4.979E-05	.0177	6.070E-06
IL	382	56	.152	.116	.0211	.0151	4.539E-06	3.445E-05	.0143	4.010E-06
NM	230	46	.337	.103	.0141	.0228	1.201E-05	1.048E-04	.0238	1.078E-05
OH	368	56	.151	.144	.0083	.0152	6.041E-06	3.598E-05	.0164	5.173E-06
NE	199	43	.397	.089	.0316	.0256	1.337E-05	1.376E-04	.0250	1.218E-05
DC	240	48	.297	.095	.0261	.0213	9.391E-06	8.929E-05	.0208	8.497E-06
HI	211	45	.349	.087	.0316	.0235	1.112E-05	1.162E-04	.0228	1.015E-05
WA	389	54	.182	.155	.0110	.0165	6.999E-06	3.804E-05	.0175	5.912E-06
OR	280	50	.264	.116	.0147	.0199	9.245E-06	7.031E-05	.0206	8.171E-06
AK	160	38	.479	.082	.0327	.0290	1.734E-05	1.930E-04	.0286	1.591E-05
DE	164	36	.524	.085	.0453	.0311	1.897E-05	2.046E-04	.0298	1.736E-05
Average					.0185	.0173	6.999E-06	6.448E-05	.0173	6.266E-06

Table E-4. Composite estimates of unit variance, using high estimate of average squared bias, by states

State	n'	beta	f	Weight	Unit variance: Direct	Unit variance: Composite	Variance of: Group average	Variance of: Local variance	Group average variance	Variance of composite
ND	144	39	.456	.387	.0268	.0188	4.028E-06	5.729E-05	.0137	1.008E-05
NV	151	39	.462	.407	.0146	.0146	4.599E-06	5.336E-05	.0146	1.047E-05
NC	368	56	.147	.802	.0070	.0071	1.550E-06	8.288E-06	.0075	5.396E-06
IA	344	51	.239	.722	.0077	.0083	2.368E-06	1.327E-05	.0098	7.099E-06
VT	145	38	.482	.390	.0185	.0162	4.767E-06	5.770E-05	.0148	1.054E-05
KY	360	56	.156	.790	.0060	.0064	1.666E-06	8.992E-06	.0079	5.680E-06
VA	364	55	.162	.790	.0075	.0076	1.643E-06	8.978E-06	.0078	5.673E-06
IN	377	56	.161	.786	.0034	.0045	1.833E-06	9.258E-06	.0085	5.798E-06
NH	147	39	.473	.367	.0335	.0209	3.889E-06	6.214E-05	.0137	9.915E-06
UT	177	37	.500	.433	.0191	.0169	5.096E-06	4.860E-05	.0152	1.077E-05
MT	150	38	.479	.226	.0350	.0262	1.035E-05	1.455E-04	.0236	1.362E-05
ME	219	46	.335	.345	.0197	.0192	6.669E-06	7.358E-05	.0189	1.162E-05
FL	360	56	.153	.598	.0167	.0147	2.658E-06	2.337E-05	.0117	8.784E-06
AR	241	51	.252	.413	.0113	.0142	4.899E-06	5.258E-05	.0162	1.065E-05
KS	257	48	.298	.397	.0243	.0199	5.460E-06	5.705E-05	.0170	1.097E-05
SD	151	39	.456	.234	.0124	.0212	1.021E-05	1.384E-04	.0239	1.358E-05
LA	373	56	.154	.610	.0137	.0131	2.897E-06	2.232E-05	.0121	8.759E-06
GA	361	56	.146	.612	.0140	.0131	2.729E-06	2.210E-05	.0118	8.681E-06
CT	358	53	.211	.541	.0074	.0110	4.400E-06	3.092E-05	.0152	9.985E-06
MO	405	56	.149	.637	.0112	.0116	3.008E-06	1.999E-05	.0123	8.511E-06
TN	366	56	.159	.599	.0095	.0109	3.236E-06	2.362E-05	.0129	9.003E-06
RI	219	44	.369	.314	.0172	.0207	1.115E-05	9.444E-05	.0222	1.456E-05
SC	363	54	.194	.518	.0099	.0129	6.529E-06	3.587E-05	.0162	1.116E-05
NY	357	56	.148	.535	.0239	.0187	4.251E-06	3.163E-05	.0127	9.962E-06
CO	288	48	.299	.398	.0091	.0159	9.381E-06	6.268E-05	.0203	1.334E-05
MI	364	56	.150	.559	.0129	.0134	5.219E-06	2.945E-05	.0141	1.021E-05
PA	365	56	.148	.526	.0273	.0201	3.830E-06	3.237E-05	.0122	9.816E-06
WI	372	56	.149	.561	.0182	.0161	4.814E-06	2.892E-05	.0134	1.002E-05
AZ	258	49	.286	.357	.0359	.0240	7.216E-06	7.094E-05	.0175	1.200E-05
MS	361	55	.176	.519	.0036	.0097	6.229E-06	3.555E-05	.0163	1.101E-05
MN	366	54	.192	.511	.0038	.0102	6.673E-06	3.713E-05	.0169	1.128E-05
MA	366	56	.149	.444	.0184	.0182	7.176E-06	4.924E-05	.0181	1.191E-05
NJ	362	56	.149	.442	.0148	.0169	7.468E-06	4.994E-05	.0185	1.208E-05
AL	367	55	.179	.419	.0255	.0213	7.184E-06	5.452E-05	.0182	1.199E-05
WV	298	51	.239	.356	.0126	.0186	9.814E-06	7.573E-05	.0220	1.368E-05
OK	278	50	.268	.324	.0067	.0181	1.075E-05	8.958E-05	.0235	1.429E-05
ID	156	37	.495	.185	.0540	.0330	1.545E-05	2.091E-04	.0283	1.743E-05
MD	363	56	.150	.441	.0130	.0163	7.651E-06	5.043E-05	.0188	1.219E-05
WY	164	39	.471	.199	.0334	.0295	1.574E-05	1.922E-04	.0285	1.772E-05
CA	387	56	.151	.447	.0245	.0205	6.534E-06	4.773E-05	.0173	1.154E-05
TX	363	56	.149	.439	.0208	.0191	6.913E-06	4.979E-05	.0177	1.178E-05
IL	382	56	.152	.515	.0211	.0178	4.539E-06	3.445E-05	.0143	1.021E-05
NM	230	46	.337	.296	.0141	.0210	1.201E-05	1.048E-04	.0238	1.514E-05
OH	368	56	.151	.514	.0083	.0122	6.041E-06	3.598E-05	.0164	1.095E-05
NE	199	43	.397	.248	.0316	.0266	1.337E-05	1.376E-04	.0250	1.604E-05
DC	240	48	.297	.317	.0261	.0225	9.391E-06	8.929E-05	.0208	1.336E-05
HI	211	45	.349	.271	.0316	.0252	1.112E-05	1.162E-04	.0228	1.444E-05
WA	389	54	.182	.507	.0110	.0142	6.999E-06	3.804E-05	.0175	1.147E-05
OR	280	50	.264	.370	.0147	.0184	9.245E-06	7.031E-05	.0206	1.330E-05
AK	160	38	.479	.204	.0327	.0295	1.734E-05	1.930E-04	.0286	1.902E-05
DE	164	36	.524	.200	.0453	.0329	1.897E-05	2.046E-04	.0298	2.031E-05
Average					.0185	.0174	6.999E-06	6.448E-05	.0173	1.153E-05

Table E-5. Pooled unit variance estimates, by states

State	n'	f	Pooled unit variance
ND	144	0.456	0.01708
NV	151	0.462	0.01724
NC	368	0.147	0.00885
IA	344	0.239	0.01130
VT	145	0.482	0.01777
KY	360	0.156	0.00908
VA	364	0.162	0.00924
IN	377	0.161	0.00922
NH	147	0.473	0.01753
UT	177	0.500	0.01826
MT	150	0.479	0.02550
ME	219	0.335	0.02008
FL	360	0.153	0.01320
AR	241	0.252	0.01692
KS	257	0.298	0.01866
SD	151	0.456	0.02463
LA	373	0.154	0.01323
GA	361	0.146	0.01295
CT	358	0.211	0.01539
MO	405	0.149	0.01306
TN	366	0.159	0.01343
RI	219	0.369	0.02686
SC	363	0.194	0.01907
NY	357	0.148	0.01705
CO	288	0.299	0.02372
MI	364	0.150	0.01712
PA	365	0.148	0.01702
WI	372	0.149	0.01706
AZ	258	0.286	0.02317
MS	361	0.176	0.01829
MN	366	0.192	0.01899
MA	366	0.149	0.02251
NJ	362	0.149	0.02250
AL	367	0.179	0.02368
WV	298	0.239	0.02598
OK	278	0.268	0.02710
ID	156	0.495	0.03592
MD	363	0.150	0.02256
WY	164	0.471	0.03499
CA	387	0.151	0.02259
TX	363	0.149	0.02251
IL	382	0.152	0.01794
NM	230	0.337	0.02543
OH	368	0.151	0.01792
NE	199	0.397	0.02785
DC	240	0.297	0.02379
HI	211	0.349	0.02592
WA	389	0.182	0.01917
OR	280	0.264	0.02249
AK	160	0.479	0.03115
DE	164	0.524	0.03296

Figure E-5. Weights for the composite estimate using zero and high squared bias

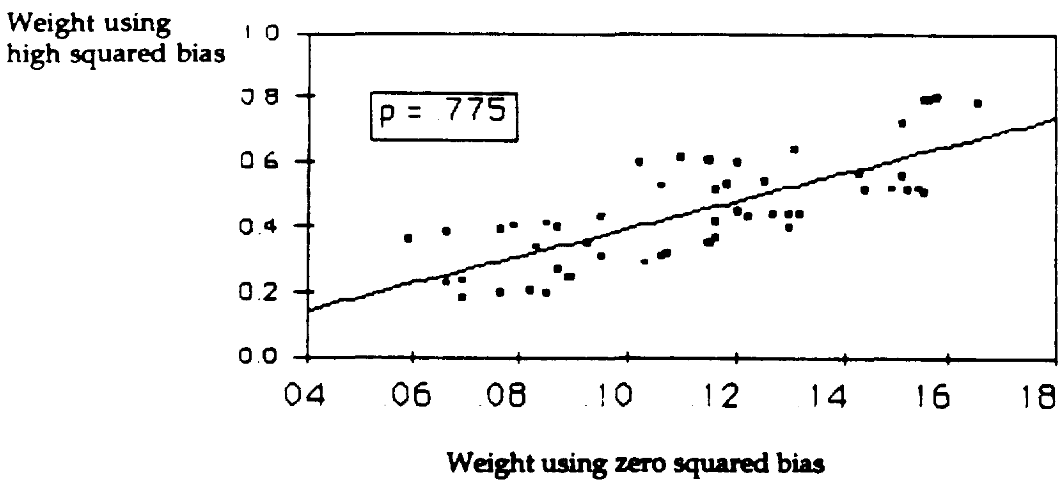


Figure E-6. Relationship of the simple pooled estimate and the composite estimate using zero squared bias

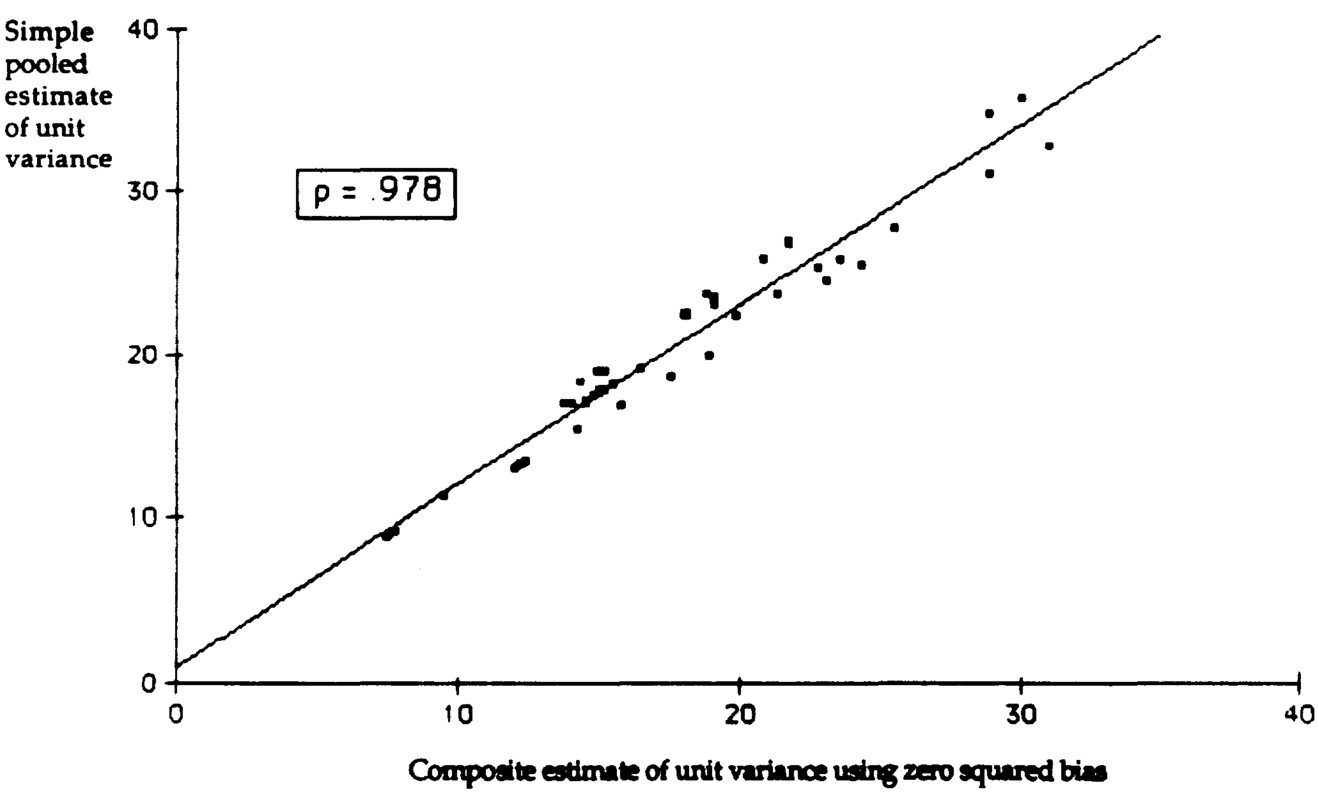
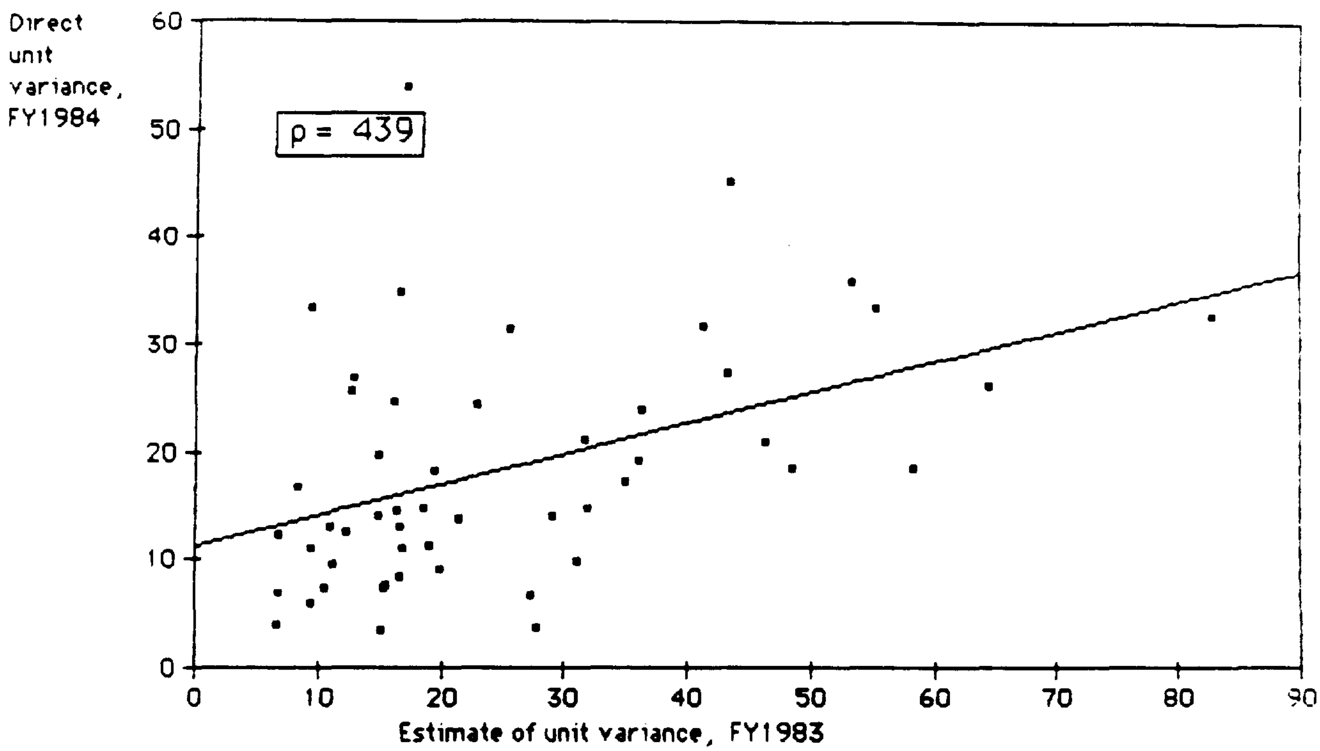


Figure E-7. Relationship of various estimates of unit variance for 1984 to the direct estimate for 1983 ($\times 10^3$)



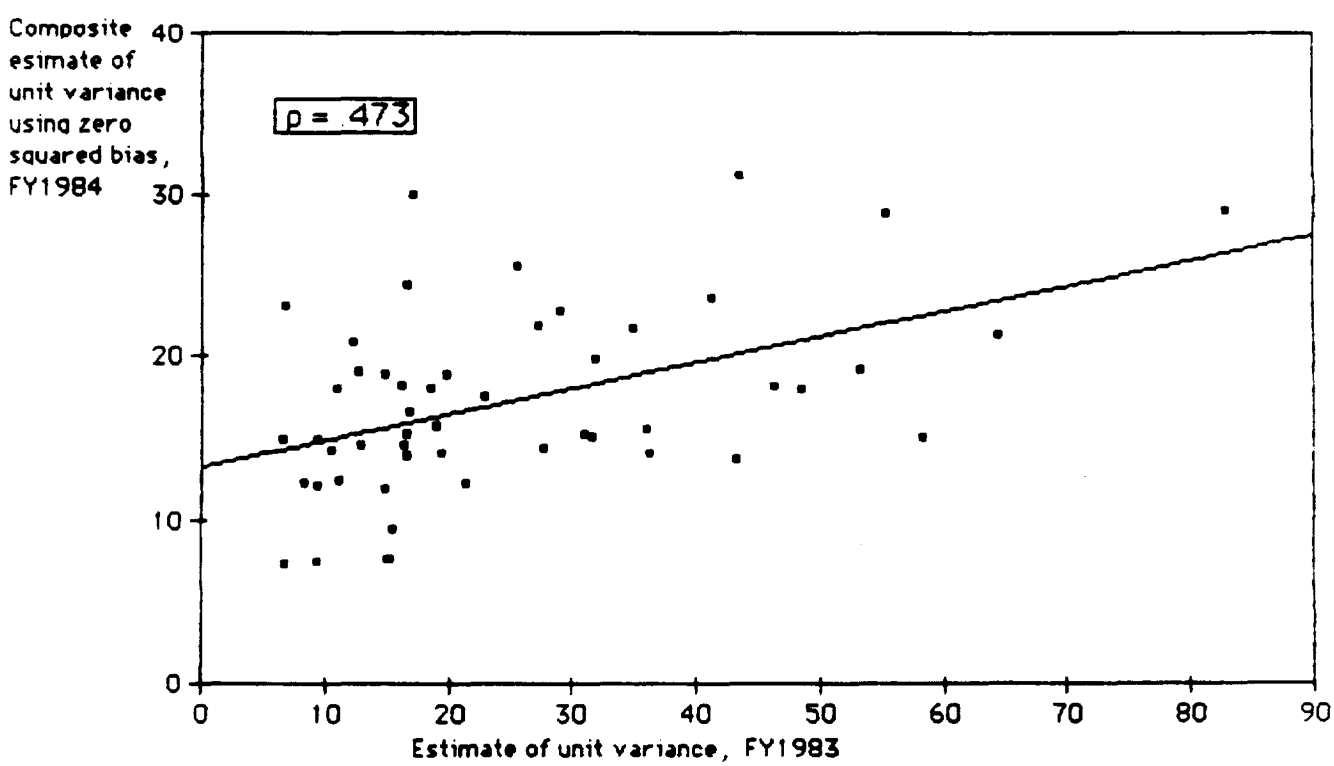
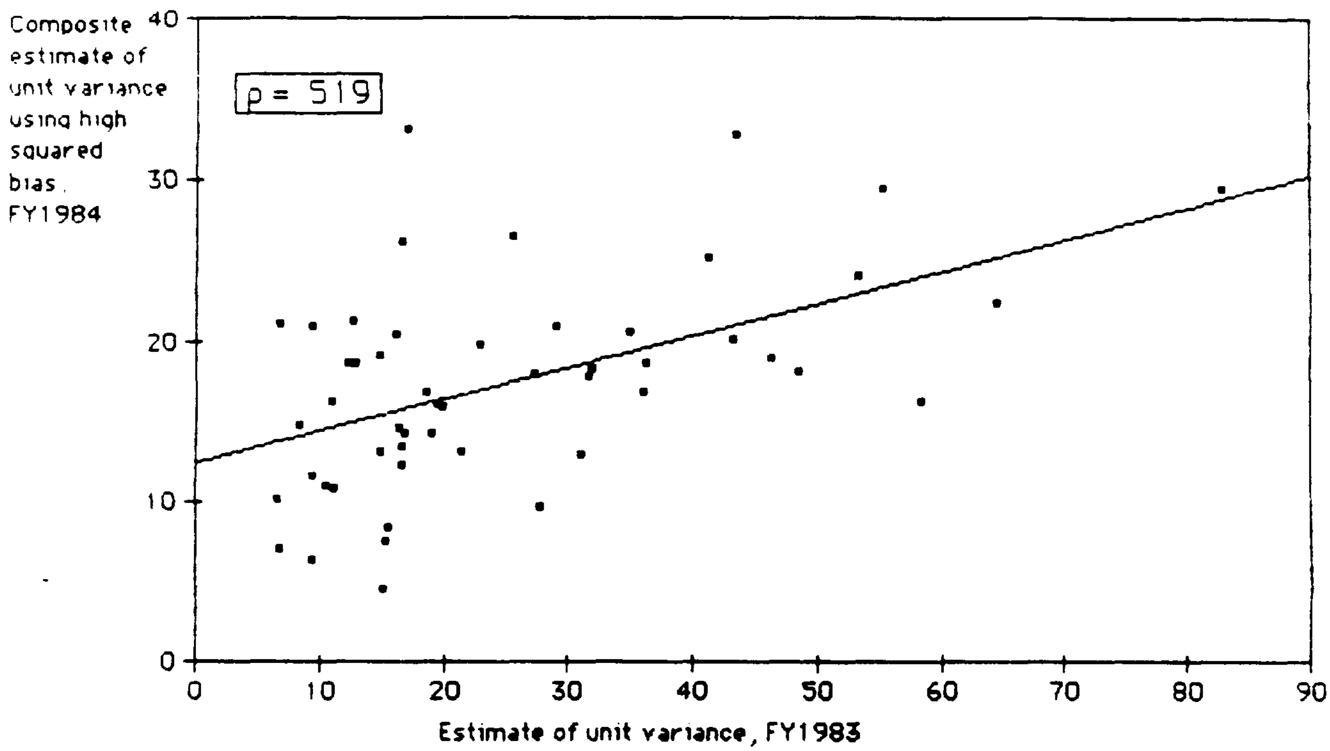
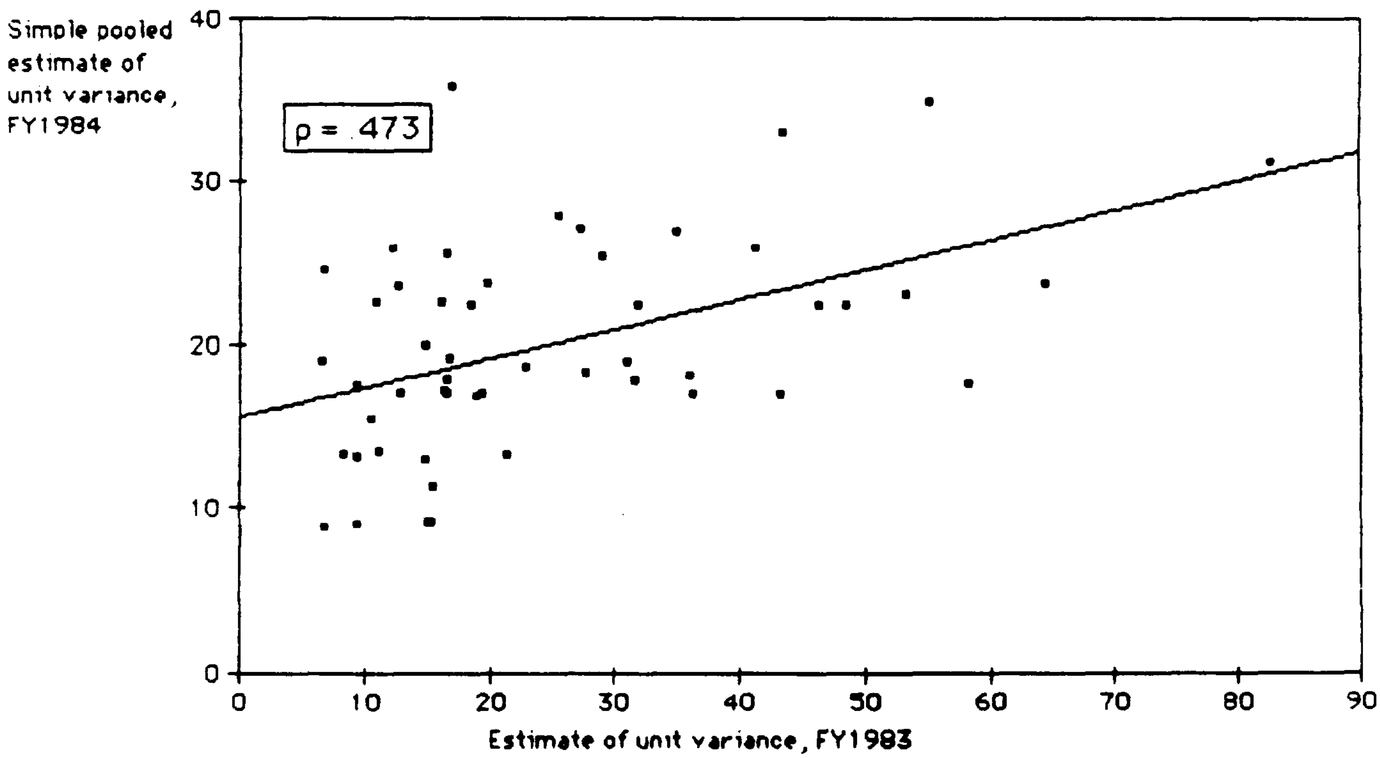


Figure E-7. Relationship of various estimates of unit variance for 1984 to the direct estimate for 1983 (× 10^3) (continued)





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APPENDIX F

OPTIMUM SAMPLE SIZE FOR DISALLOWANCES BASED ON POINT ESTIMATES

For the purpose of this appendix, we may define the optimum sample size as that which minimizes the cost. But the cost can be defined in more than one way. We shall define the expected cost from the Federal point of view as the Federal share of the cost of review of the state sample plus the cost of review and processing the Federal subsample minus the expected value of the disallowance assessed. We shall define the expected cost from a state's point of view as its cost of processing the state sample plus the expected value of the disallowance assessed.

Let us denote

U	=	the Federal contribution for the time period;
k	=	proportion of the cost that is borne by the state;
n	=	size of the state sample;
n'	=	size of the Federal subsample;
c_0	=	state share of the state cost per case in the state sample;
c_1	=	Federal share of the state cost per case in the state sample;
c_2	=	Federal cost per case in the Federal subsample;
r	=	estimated payment error rate; and
R	=	$E(r)$, the expectation of r.

We consider, first, the problem of minimizing the variance (thus maximizing the precision) of the estimated payment error rate, for a fixed Federal cost K defined by $c_1n + c_2n'$. The minimizing values of n and n' are obtained by setting equal to zero the partial derivatives of the function

$$\psi = \sigma_r^2 - \lambda(K - c_1 n - c_2 n')$$

and solving the resulting equations for λ, n and n'. This gives the optimum subsampling fraction f = n'/n as

$$f^2 = \{(1-\rho^2)/\rho^2\}\{c_1/c_2\}.$$

The optimizing sample sizes are then

$$n = K/(c_1 + fc_2)$$

$$n' = fn.$$

Present plans call for annual samples of n=2400 and n'=360 in large states. It has been estimated that c_1=\$130 and c_2=\$330, which gives rise to the value K=\$430,800. The values that would minimize the variance for that cost would be n=1667 and n'=649.

We now suppose that a portion of the Federal contribution U to a state is withheld when the point estimate of the payment error rate, r, exceeds .03, and that then the disallowance is the fraction of the Federal contribution equal to the excess of r over the tolerance level .03. Let

$$\xi = (r - .03)$$

$$\mu = E(\xi) = (R - .03)\,U$$

$$\sigma^2 = \sigma_\xi^2 = U^2\sigma_r^2.$$

The disallowance is defined by

$$D = \begin{cases} \xi & \text{if } r > .03 \\ 0, & \text{otherwise.} \end{cases}$$

It can be shown (see note at the end of this Appendix) that, since ξ is approximately normally distributed, the expected value of the disallowance is approximately

$$E(D) = (\sigma/\sqrt{2\pi})\exp(-\mu^2/2\sigma^2) + (\mu/\sqrt{2\pi})\int_{-\mu/\sigma}^{\infty} \exp(-t^2/2)\,dt.$$

This expression can be evaluated, given the values of σ (which is a function of n, n', and certain other parameters) and of μ (which is a function of R and U).

The expected value of the gain to the Federal government is

$$G = E(D) - c_1 n - c_2 n'.$$

We pose the question: given that it is required to attain a variance σ_r^2 of the estimated payment error rate, is it possible to choose a state sample size n that maximizes G? We have

$$\sigma_r^2 = \{\sigma_x^2/\bar{T}^2 n'\}\{1 - (1-n'/n)\rho^2\};$$

σ_x^2 = variance of the payment error finding by the Federal review;

$\bar{T}$ = average AFDC payment[1]; and

ρ = correlation between the Federal and state findings.

To attain a given variance σ_r^2 given the sample size n, we must have

$$n' = (1-\rho^2)/(\bar{T}^2\sigma_r^2/\sigma_x^2 - \rho^2/n).$$

Since $n' \leq n$, we must satisfy the inequality

[1]We have used $\bar{T}$ (which is a constant) in the estimate rather than the estimate from the state sample, in order to simplify this analysis.

$$n \geq \sigma_x^2 / \bar{T}^2 \sigma_r^2 .$$

Thus, for example, if the standard error is to be at most σ_r=.01, and if the ratio $\sigma_x/\bar{T}$=.2, then the state sample size must be at least n=400. The Federal subsample size would then have to be n'=289 if ρ=.85. If n were increased to 2400, the desired standard error would be attained with n'=127.

For values of n satisfying the conditions stated above, we now examine the properties of G as a function of n for a fixed value of σ_r. We have

$$dG/dn = -c_1 - c_2\, dn'/dn$$

$$= -c_1 + c_2(1-\rho^2)\rho^2/(n\bar{T}^2\sigma_r^2/\sigma_x^2-\rho^2)^2 .$$

Table F-1 gives the values of this derivative for c_1=130, c_2=330 and several values of the other parameters. An entry of zero in the table indicates that the specified standard error cannot be attained with the associated value of n. The table shows that once the state sample is of sufficient size to yield the desired standard error, increasing the size of the Federal subsample will only reduce the expected value of the Federal gain.

We now also examine the effect on the expected value of the Federal gain that would result from varying the desired standard error. The derivative of E(D) with respect to σ is

$$(1/\sqrt{2\pi}) \exp(-\mu^2/2\sigma^2)$$

which is always positive. For a fixed n, we have dD/dn'=(dD/dσ)(dσ/dn'). Since dσ/dn' is clearly negative, so is dD/dn'. Thus, the expected Federal gain is a *decreasing* function of the Federal sample size, for any given size of the state sample. It follows that to maximize the expected value of the Federal gain, given the state sample, the Federal sample should be as small as possible. Similarly, to maximize

the Federal gain, given the size of the Federal subsample, the state sample should be as small as possible. We conclude that from the point of view of maximizing the expected value of the Federal gain, there is no optimum choice of the sample sizes.

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TECHNICAL NOTE FOR APPENDIX F

Theorem: Let ξ be normally distributed with mean μ and variance σ^2, and let D be the random variable defined by

$$D = \begin{cases} \xi & \text{if } \xi > 0 \\ 0, & \text{if } \xi \le 0. \end{cases}$$

Then the mathematical expectation of D is

$$ED = (\sigma/\sqrt{2\pi})\exp(-\mu^2/2\sigma^2) + (\mu/\sqrt{2\pi})\int_{-\mu/\sigma}^{\infty} \exp(-t^2/2)\,.$$

Proof:

$$ED = \text{Prob}(\xi \le 0) \times 0 + \text{Prob}(\xi > 0) \times E(\xi \mid \xi > 0)$$

$$= (1/\sqrt{2\pi}\,\sigma)\int_{0}^{\infty} x \exp\{-(x-\mu^2/2\sigma^2)\}\,dx.$$

Under the transformation $t=(x-\mu)/\sigma$ we get

$$ED = (1/\sqrt{2\pi})\int_{-\mu/\sigma}^{\infty} (\sigma t + \mu)\exp(-t^2/2)\,dt$$

$$= (\sigma/\sqrt{2\pi})\int_{-\mu/\sigma}^{\infty} t\exp(-t^2/2)\,dt + (\mu/\sqrt{2\pi})\int_{-\mu/\sigma}^{\infty} \exp(-t^2/2)\,dt$$

$$= (\sigma/\sqrt{2\pi})\exp(-\mu^2/2\sigma^2) + (\mu/\sqrt{2\pi})\int_{-\mu/\sigma}^{\infty} \exp(-t^2/2)\,dt$$

which was to be proved.

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Table F-1. Slope of the Federal gain function of the state sample size

rho=	0.85	T-bar=	300	S(x)=	60
n	Standard error of the estimated payment error rate				
	0.005	0.01	0.015	0.02	0.025
100	0	0	0	-129.7616	-129.9212
200	0	0	-129.8356	-129.9482	-129.9725
300	0	0	-129.9314	-129.9709	-129.9833
400	0	-129.7616	-129.9567	-129.9798	-129.988
500	0	-129.8746	-129.9683	-129.9845	-129.9907
600	0	-129.9149	-129.9751	-129.9875	-129.9924
700	0	-129.9356	-129.9794	-129.9895	-129.9935
800	0	-129.9482	-129.9825	-129.9909	-129.9944
900	0	-129.9567	-129.9848	-129.992	-129.995
1000	0	-129.9628	-129.9865	-129.9929	-129.9956
1100	0	-129.9674	-129.9879	-129.9936	-129.996
1200	0	-129.9709	-129.989	-129.9941	-129.9963
1300	0	-129.9738	-129.99	-129.9946	-129.9966
1400	0	-129.9762	-129.9907	-129.995	-129.9969
1500	0	-129.9781	-129.9914	-129.9954	-129.9971
1600	-129.7616	-129.9798	-129.992	-129.9957	-129.9973
1700	-129.8054	-129.9812	-129.9925	-129.9959	-129.9974
1800	-129.8356	-129.9825	-129.993	-129.9962	-129.9976
1900	-129.8577	-129.9836	-129.9934	-129.9964	-129.9977
2000	-129.8746	-129.9845	-129.9937	-129.9966	-129.9978
2100	-129.8879	-129.9854	-129.994	-129.9967	-129.9979
2200	-129.8986	-129.9862	-129.9943	-129.9969	-129.998
2300	-129.9075	-129.9868	-129.9946	-129.997	-129.9981
2400	-129.9149	-129.9875	-129.9948	-129.9972	-129.9982
2500	-129.9212	-129.988	-129.995	-129.9973	-129.9983
2600	-129.9267	-129.9885	-129.9952	-129.9974	-129.9983
2700	-129.9314	-129.989	-129.9954	-129.9975	-129.9984
2800	-129.9356	-129.9895	-129.9956	-129.9976	-129.9985
2900	-129.9393	-129.9899	-129.9958	-129.9977	-129.9985
3000	-129.9426	-129.9902	-129.9959	-129.9977	-129.9986

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Table F-1. Slope of the Federal gain function of the state sample size (continued)

rho= 0.85 T-bar= 300 S(x)= 40

n	Standard error of the estimated payment error rate				
	0.005	0.01	0.015	0.02	0.025
100	0	0	-129.8782	-129.9567	-129.9763
200	0	-129.8356	-129.9634	-129.9825	-129.9895
300	0	-129.9314	-129.9785	-129.989	-129.9933
400	0	-129.9567	-129.9848	-129.992	-129.995
500	0	-129.9683	-129.9882	-129.9937	-129.9961
600	0	-129.9751	-129.9904	-129.9948	-129.9968
700	0	-129.9794	-129.9919	-129.9956	-129.9972
800	-129.8356	-129.9825	-129.993	-129.9962	-129.9976
900	-129.8782	-129.9848	-129.9938	-129.9966	-129.9979
1000	-129.9032	-129.9865	-129.9945	-129.997	-129.9981
1100	-129.9197	-129.9879	-129.995	-129.9972	-129.9983
1200	-129.9314	-129.989	-129.9954	-129.9975	-129.9984
1300	-129.9402	-129.99	-129.9958	-129.9977	-129.9985
1400	-129.9469	-129.9907	-129.9961	-129.9979	-129.9986
1500	-129.9523	-129.9914	-129.9964	-129.998	-129.9987
1600	-129.9567	-129.992	-129.9966	-129.9981	-129.9988
1700	-129.9603	-129.9925	-129.9968	-129.9982	-129.9989
1800	-129.9634	-129.993	-129.997	-129.9983	-129.9989
1900	-129.9661	-129.9934	-129.9972	-129.9984	-129.999
2000	-129.9683	-129.9937	-129.9973	-129.9985	-129.999
2100	-129.9703	-129.994	-129.9974	-129.9986	-129.9991
2200	-129.9721	-129.9943	-129.9976	-129.9986	-129.9991
2300	-129.9737	-129.9946	-129.9977	-129.9987	-129.9992
2400	-129.9751	-129.9948	-129.9978	-129.9988	-129.9992
2500	-129.9763	-129.995	-129.9979	-129.9988	-129.9992
2600	-129.9774	-129.9952	-129.9979	-129.9989	-129.9993
2700	-129.9785	-129.9954	-129.998	-129.9989	-129.9993
2800	-129.9794	-129.9956	-129.9981	-129.9989	-129.9993
2900	-129.9803	-129.9958	-129.9982	-129.999	-129.9993
3000	-129.9811	-129.9959	-129.9982	-129.999	-129.9994

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Table F-1. Slope of the Federal gain function of the state sample size (continued)

rho=	0.85	T-bar=	300	S(x)=	100
n	Standard error of the estimated payment error rate				
	0.005	0.01	0.015	0.02	0.025
100	0	0	0	0	0
200	0	0	0	0	-129.8356
300	0	0	0	-129.8149	-129.9314
400	0	0	0	-129.9078	-129.9567
500	0	0	-129.7719	-129.9386	-129.9683
600	0	0	-129.8657	-129.954	-129.9751
700	0	0	-129.9048	-129.9632	-129.9794
800	0	0	-129.9263	-129.9693	-129.9825
900	0	0	-129.9399	-129.9737	-129.9848
1000	0	0	-129.9492	-129.977	-129.9865
1100	0	0	-129.956	-129.9796	-129.9879
1200	0	-129.8149	-129.9613	-129.9816	-129.989
1300	0	-129.8521	-129.9654	-129.9833	-129.99
1400	0	-129.8769	-129.9687	-129.9847	-129.9907
1500	0	-129.8946	-129.9714	-129.9859	-129.9914
1600	0	-129.9078	-129.9737	-129.9869	-129.992
1700	0	-129.9181	-129.9757	-129.9877	-129.9925
1800	0	-129.9263	-129.9774	-129.9885	-129.993
1900	0	-129.933	-129.9788	-129.9892	-129.9934
2000	0	-129.9386	-129.9801	-129.9898	-129.9937
2100	0	-129.9433	-129.9813	-129.9903	-129.994
2200	0	-129.9474	-129.9823	-129.9908	-129.9943
2300	0	-129.9509	-129.9832	-129.9912	-129.9946
2400	0	-129.954	-129.984	-129.9916	-129.9948
2500	0	-129.9567	-129.9848	-129.992	-129.995
2600	0	-129.9591	-129.9854	-129.9923	-129.9952
2700	0	-129.9613	-129.9861	-129.9926	-129.9954
2800	0	-129.9632	-129.9866	-129.9929	-129.9956
2900	0	-129.9649	-129.9872	-129.9932	-129.9958
3000	0	-129.9665	-129.9876	-129.9934	-129.9959

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Table F-1. Slope of the Federal gain function of the state sample size (continued)

rho=	0.9	T-bar=	300	S(x)=	60
n	Standard error of the estimated payment error rate				
	0.005	0.01	0.015	0.02	0.025
100	0	0	0	-129.7327	-129.9325
200	0	0	-129.8388	-129.9573	-129.9781
300	0	0	-129.9421	-129.9768	-129.9869
400	0	-129.7327	-129.9647	-129.9841	-129.9907
500	0	-129.8846	-129.9746	-129.9879	-129.9927
600	0	-129.9264	-129.9802	-129.9902	-129.9941
700	0	-129.946	-129.9838	-129.9918	-129.995
800	0	-129.9573	-129.9862	-129.9929	-129.9957
900	0	-129.9647	-129.9881	-129.9938	-129.9962
1000	0	-129.9699	-129.9895	-129.9945	-129.9966
1100	0	-129.9738	-129.9906	-129.995	-129.9969
1200	0	-129.9768	-129.9915	-129.9955	-129.9972
1300	0	-129.9792	-129.9922	-129.9958	-129.9974
1400	0	-129.9811	-129.9928	-129.9961	-129.9976
1500	0	-129.9827	-129.9933	-129.9964	-129.9978
1600	-129.7327	-129.9841	-129.9938	-129.9967	-129.9979
1700	-129.7989	-129.9852	-129.9942	-129.9969	-129.998
1800	-129.8388	-129.9862	-129.9945	-129.997	-129.9981
1900	-129.8655	-129.9871	-129.9949	-129.9972	-129.9982
2000	-129.8846	-129.9879	-129.9951	-129.9974	-129.9983
2100	-129.8989	-129.9886	-129.9954	-129.9975	-129.9984
2200	-129.9101	-129.9892	-129.9956	-129.9976	-129.9985
2300	-129.9191	-129.9897	-129.9958	-129.9977	-129.9986
2400	-129.9264	-129.9902	-129.996	-129.9978	-129.9986
2500	-129.9325	-129.9907	-129.9962	-129.9979	-129.9987
2600	-129.9377	-129.9911	-129.9963	-129.998	-129.9987
2700	-129.9421	-129.9915	-129.9965	-129.9981	-129.9988
2800	-129.946	-129.9918	-129.9966	-129.9981	-129.9988
2900	-129.9493	-129.9921	-129.9967	-129.9982	-129.9989
3000	-129.9523	-129.9924	-129.9968	-129.9983	-129.9989

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Table F-1. Slope of the Federal gain function of the state sample size (continued)

rho=	0.9	T-bar=	300	S(x)=	40
n	Standard error of the estimated payment error rate				
	0.005	0.01	0.015	0.02	0.025
100	0	0	-129.8885	-129.9647	-129.9812
200	0	-129.8388	-129.9705	-129.9862	-129.9918
300	0	-129.9421	-129.983	-129.9915	-129.9948
400	0	-129.9647	-129.9881	-129.9938	-129.9962
500	0	-129.9746	-129.9908	-129.9951	-129.997
600	0	-129.9802	-129.9925	-129.996	-129.9975
700	0	-129.9838	-129.9937	-129.9966	-129.9979
800	-129.8388	-129.9862	-129.9945	-129.997	-129.9981
900	-129.8885	-129.9881	-129.9952	-129.9974	-129.9984
1000	-129.9148	-129.9895	-129.9957	-129.9977	-129.9985
1100	-129.9311	-129.9906	-129.9961	-129.9979	-129.9987
1200	-129.9421	-129.9915	-129.9965	-129.9981	-129.9988
1300	-129.9501	-129.9922	-129.9968	-129.9982	-129.9989
1400	-129.9562	-129.9928	-129.997	-129.9983	-129.999
1500	-129.9609	-129.9933	-129.9972	-129.9985	-129.999
1600	-129.9647	-129.9938	-129.9974	-129.9986	-129.9991
1700	-129.9679	-129.9942	-129.9975	-129.9986	-129.9991
1800	-129.9705	-129.9945	-129.9977	-129.9987	-129.9992
1900	-129.9727	-129.9949	-129.9978	-129.9988	-129.9992
2000	-129.9746	-129.9951	-129.9979	-129.9989	-129.9993
2100	-129.9763	-129.9954	-129.998	-129.9989	-129.9993
2200	-129.9778	-129.9956	-129.9981	-129.999	-129.9993
2300	-129.9791	-129.9958	-129.9982	-129.999	-129.9994
2400	-129.9802	-129.996	-129.9983	-129.999	-129.9994
2500	-129.9812	-129.9962	-129.9984	-129.9991	-129.9994
2600	-129.9822	-129.9963	-129.9984	-129.9991	-129.9994
2700	-129.983	-129.9965	-129.9985	-129.9992	-129.9995
2800	-129.9838	-129.9966	-129.9985	-129.9992	-129.9995
2900	-129.9845	-129.9967	-129.9986	-129.9992	-129.9995
3000	-129.9851	-129.9968	-129.9986	-129.9992	-129.9995

Table F-1. Slope of the Federal gain function of the state sample size (continued)

rho=	0.9	T-bar=	300	S(x)=	100
n	Standard error of the estimated payment error rate				
	0.005	0.01	0.015	0.02	[illegible]25
100	0	0	0	0	0
200	0	0	0	0	-129.8388
300	0	0	0	-129.8119	-129.9421
400	0	0	0	-129.9194	-129.9647
500	0	0	-129.7492	-129.9487	-129.9746
600	0	0	-129.8746	-129.9624	-129.9802
700	0	0	-129.9164	-129.9703	-129.9838
800	0	0	-129.9373	-129.9755	-129.9862
900	0	0	-129.9498	-129.9791	-129.9881
1000	0	0	-129.9582	-129.9818	-129.9895
1100	0	0	-129.9642	-129.9839	-129.9906
1200	0	-129.8119	-129.9687	-129.9855	-129.9915
1300	0	-129.8589	-129.9721	-129.9869	-129.9922
1400	0	-129.8871	-129.9749	-129.988	-129.9928
1500	0	-129.906	-129.9772	-129.9889	-129.9933
1600	0	-129.9194	-129.9791	-129.9897	-129.9938
1700	0	-129.9295	-129.9807	-129.9904	-129.9942
1800	0	-129.9373	-129.9821	-129.991	-129.9945
1900	0	-129.9436	-129.9833	-129.9916	-129.9949
2000	0	-129.9487	-129.9843	-129.9921	-129.9951
2100	0	-129.953	-129.9852	-129.9925	-129.9954
2200	0	-129.9566	-129.9861	-129.9929	-129.9956
2300	0	-129.9597	-129.9868	-129.9932	-129.9958
2400	0	-129.9624	-129.9875	-129.9935	-129.996
2500	0	-129.9647	-129.9881	-129.9938	-129.9962
2600	0	-129.9668	-129.9886	-129.9941	-129.9963
2700	0	-129.9687	-129.9891	-129.9943	-129.9965
2800	0	-129.9703	-129.9896	-129.9945	-129.9966
2900	0	-129.9718	-129.99	-129.9947	-129.9967
3000	0	-129.9731	-129.9904	-129.9949	-129.9968

Table F-1. Slope of the Federal gain function of the state sample size (continued)

rho=	0.8	T-bar=	300	S(x)=	60
n	Standard error of the estimated payment error rate				
	0.005	0.01	0.015	0.02	0.025
100	0	0	0	-129.7888	-129.9176
200	0	0	-129.8432	-129.9441	-129.9694
300	0	0	-129.9274	-129.9678	-129.9812
400	0	-129.7888	-129.9528	-129.9774	-129.9864
500	0	-129.8754	-129.965	-129.9826	-129.9894
600	0	-129.9116	-129.9722	-129.9858	-129.9913
700	0	-129.9315	-129.9769	-129.988	-129.9926
800	0	-129.9441	-129.9803	-129.9897	-129.9936
900	0	-129.9528	-129.9828	-129.9909	-129.9943
1000	0	-129.9591	-129.9847	-129.9919	-129.9949
1100	0	-129.964	-129.9863	-129.9927	-129.9954
1200	0	-129.9678	-129.9876	-129.9933	-129.9958
1300	0	-129.9709	-129.9886	-129.9938	-129.9961
1400	0	-129.9734	-129.9895	-129.9943	-129.9964
1500	0	-129.9756	-129.9902	-129.9947	-129.9967
1600	-129.7888	-129.9774	-129.9909	-129.9951	-129.9969
1700	-129.82	-129.9789	-129.9915	-129.9954	-129.9971
1800	-129.8432	-129.9803	-129.992	-129.9956	-129.9972
1900	-129.8611	-129.9815	-129.9924	-129.9959	-129.9974
2000	-129.8754	-129.9826	-129.9928	-129.9961	-129.9975
2100	-129.8869	-129.9835	-129.9932	-129.9963	-129.9976
2200	-129.8966	-129.9844	-129.9935	-129.9964	-129.9977
2300	-129.9047	-129.9851	-129.9938	-129.9966	-129.9978
2400	-129.9116	-129.9858	-129.9941	-129.9967	-129.9979
2500	-129.9176	-129.9864	-129.9943	-129.9969	-129.998
2600	-129.9228	-129.987	-129.9946	-129.997	-129.9981
2700	-129.9274	-129.9876	-129.9948	-129.9971	-129.9982
2800	-129.9315	-129.988	-129.995	-129.9972	-129.9982
2900	-129.9352	-129.9885	-129.9951	-129.9973	-129.9983
3000	-129.9384	-129.9889	-129.9953	-129.9974	-129.9984

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Table F-1. Slope of the Federal gain function of the state sample size (continued)

rho=	0.8	T-bar=	300	S(x)=	40
n	Standard error of the estimated payment error rate				
	0.005	0.01	0.015	0.02	0.025
100	0	0	-129.8785	-129.9528	-129.9736
200	0	-129.8432	-129.9598	-129.9803	-129.9881
300	0	-129.9274	-129.9759	-129.9876	-129.9923
400	0	-129.9528	-129.9828	-129.9909	-129.9943
500	0	-129.965	-129.9866	-129.9928	-129.9955
600	0	-129.9722	-129.9891	-129.9941	-129.9963
700	0	-129.9769	-129.9907	-129.995	-129.9968
800	-129.8432	-129.9803	-129.992	-129.9956	-129.9972
900	-129.8785	-129.9828	-129.9929	-129.9961	-129.9975
1000	-129.9008	-129.9847	-129.9937	-129.9965	-129.9978
1100	-129.9162	-129.9863	-129.9943	-129.9968	-129.998
1200	-129.9274	-129.9876	-129.9948	-129.9971	-129.9982
1300	-129.936	-129.9886	-129.9952	-129.9973	-129.9983
1400	-129.9428	-129.9895	-129.9955	-129.9975	-129.9984
1500	-129.9483	-129.9902	-129.9959	-129.9977	-129.9985
1600	-129.9528	-129.9909	-129.9961	-129.9978	-129.9986
1700	-129.9566	-129.9915	-129.9964	-129.998	-129.9987
1800	-129.9598	-129.992	-129.9966	-129.9981	-129.9988
1900	-129.9626	-129.9924	-129.9968	-129.9982	-129.9989
2000	-129.965	-129.9928	-129.9969	-129.9983	-129.9989
2100	-129.9671	-129.9932	-129.9971	-129.9984	-129.999
2200	-129.969	-129.9935	-129.9972	-129.9984	-129.999
2300	-129.9707	-129.9938	-129.9973	-129.9985	-129.9991
2400	-129.9722	-129.9941	-129.9974	-129.9986	-129.9991
2500	-129.9736	-129.9943	-129.9975	-129.9986	-129.9991
2600	-129.9748	-129.9946	-129.9976	-129.9987	-129.9992
2700	-129.9759	-129.9948	-129.9977	-129.9987	-129.9992
2800	-129.9769	-129.995	-129.9978	-129.9988	-129.9992
2900	-129.9779	-129.9951	-129.9979	-129.9988	-129.9992
3000	-129.9788	-129.9953	-129.998	-129.9989	-129.9993

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Table F-1. Slope of the Federal gain function of the state sample size (continued)

rho=	0.8	T-bar=	300	S(x)=	100
n	Standard error of the estimated payment error rate				
	0.005	0.01	0.015	0.02	0.025
100	0	0	0	0	0
200	0	0	0	0	-129.8432
300	0	0	0	-129.8272	-129.9274
400	0	0	0	-129.905	-129.9528
500	0	0	-129.7959	-129.9345	-129.965
600	0	0	-129.8678	-129.95	-129.9722
700	0	0	-129.9022	-129.9596	-129.9769
800	0	0	-129.9224	-129.9661	-129.9803
900	0	0	-129.9357	-129.9708	-129.9828
1000	0	0	-129.9451	-129.9743	-129.9847
1100	0	0	-129.9521	-129.9771	-129.9863
1200	0	-129.8272	-129.9575	-129.9793	-129.9876
1300	0	-129.8565	-129.9618	-129.9812	-129.9886
1400	0	-129.8774	-129.9654	-129.9827	-129.9895
1500	0	-129.8929	-129.9683	-129.984	-129.9902
1600	0	-129.905	-129.9708	-129.9852	-129.9909
1700	0	-129.9146	-129.9729	-129.9861	-129.9915
1800	0	-129.9224	-129.9747	-129.987	-129.992
1900	0	-129.9289	-129.9763	-129.9877	-129.9924
2000	0	-129.9345	-129.9777	-129.9884	-129.9928
2100	0	-129.9392	-129.979	-129.989	-129.9932
2200	0	-129.9433	-129.9801	-129.9896	-129.9935
2300	0	-129.9468	-129.9811	-129.99	-129.9938
2400	0	-129.95	-129.982	-129.9905	-129.9941
2500	0	-129.9528	-129.9828	-129.9909	-129.9943
2600	0	-129.9553	-129.9836	-129.9913	-129.9946
2700	0	-129.9575	-129.9843	-129.9916	-129.9948
2800	0	-129.9596	-129.9849	-129.9919	-129.995
2900	0	-129.9614	-129.9855	-129.9922	-129.9951
3000	0	-129.9631	-129.986	-129.9925	-129.9953

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APPENDIX G

OPTIMUM SAMPLE SIZE FOR DISALLOWANCES BASED ON LOWER CONFIDENCE BOUNDS

In this appendix, we suppose that a portion of the Federal contribution is withheld when the *lower bound* of the nominal (two-sided) 90 percent (or 95 percent) confidence interval for the payment error rate exceeds .03, and that then the disallowance is the fraction of the Federal contribution equal to the excess of the lower bound over the tolerance level .03. We use the same notation as in Appendix F and we also denote

s_r = the estimated standard error of r

$$\ell = r - 1.645 s_r .$$

The disallowance D is then given by

$$D = \begin{cases} (\ell - .03)U & \text{if } \ell > .03 \\ 0, & \text{otherwise.} \end{cases}$$

For a sample that is sufficiently large, ℓ is approximately normally distributed, with mean

$$\mu_\ell = R - 1.645\sigma_r$$

and variance

$$\sigma_\ell^2 = \sigma_r^2 + (1.645)^2\sigma_{s_r}^2 - 2 \times 1.645\rho_{r,s_r}\sigma_r\sigma_{s_r} .$$

From the theorem proved in Appendix F, the expected value of D is given by

$$\sqrt{2\pi}\,E(D) = \sigma_\ell \exp(-\mu_\ell^2/2\sigma_\ell^2) + (\mu_\ell - .03)\int_{-\mu_\ell/\sigma_\ell}^{\infty} \exp(-t^2/2)\,dt.$$

As in Appendix F, the expected value of the gain to the Federal government is

$$G = E(D) - c_1 n - c_2 n'$$

but the value of E(D) is different than in the context of Appendix F.

We now ask whether there are sample sizes n and n' which maximize the expected value G of the Federal gain. As before,

$$\partial E(D)/\partial\sigma_\ell > 0$$

and

$$\partial G/\partial n = (\partial E(D)/\partial\sigma_\ell)(\partial\sigma_\ell/\partial n) - c_1.$$

But

$$\partial\sigma_\ell/\partial n = (1/2\sigma_\ell)\,\partial\sigma_\ell/\partial n$$

$$\cong (1/2\sigma_\ell)\,[\partial\sigma_r^2/\partial n + 2.706\,\partial\sigma_{s_r}^2 - 3.29\,\rho_{r,s_r}$$

$$\times\ \{\sigma_r(\partial\sigma_{s_r}/\partial n) + \sigma_{s_r}(\partial\sigma_r/\partial n)\}]$$

since ρ_{r,s_r} is insensitive to variation in n. Now, since

$$\partial\sigma_{s_r}/\partial n = (1/2\sigma_{s_r})(\partial\sigma_{s_r}^2/\partial n)$$

and

$$\partial\sigma_r/\partial n = (1/2\sigma_r)(\partial\sigma_r^2/\partial n),$$

we have

$$\partial\sigma_\ell/\partial n \cong (1/2\sigma_\ell)\,[\partial\sigma_r^2/\partial n + 2.706\partial\sigma_{s_r}/\partial n$$

$$- 1.645\{(\sigma_r/\sigma_{s_r})(\partial\sigma_{s_r}^2/\partial n + (\sigma_{s_r}/\sigma_r)(\partial\sigma_r^2/\partial n\}]$$

$$\cong (1/2\sigma_\ell)\,[(1-1.645\sigma_{s_r}/\sigma_r)\partial\sigma_r^2/\partial n$$

$$+ (2.706 - 1.645\sigma_r)/\sigma_{s_r}\,\partial\sigma_{s_r}^2/\partial n]\,.$$

This expression is difficult to evaluate analytically. It may be positive for some values of n and negative for others. We are able, however, to calculate E(D) and therefore E(G) for given values of n and n'. We have calculated the expected Federal gain for three values of the annual Federal dollar amount of contribution (20, 50, and 300 million dollars), for four levels of the population payment error rate R (.04, .05, .06, and .07), and for three levels of the unit standard deviation of the overpayment error σ_x (30, 50, and 70). These assumed values cover a reasonable range of the observed values of the parameters. For Population A, the value of R is .07297 and the value of σ_x is about 70. The unit costs assumed are

c_1 = \$130 = one-half of the cost of the state QC per case in 1982; and

c_2 = \$330 = unit cost per case of the Federal review in 1982.

The assumed values for the remaining parameters are:

$n'/n = .15$

$\rho_{xy} = .9$

$\rho_{r,s_r} = .8.$

These are reasonable values according to the available data for the year ending September 30, 1982, and for the three test populations that we constructed.

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For the above values of the parameters and for Federal subsample sizes up to n'=500, Figures G-1 through G-3 show the expected Federal gain as a proportion of the Federal contribution. The portions of the curves for extremely small sample sizes should be disregarded, for the approximations used in the mathematical development are not acceptable for such small sample sizes.

It will be seen from Table 3-3 in Chapter 3 of this report, and from Figures G-1 through G-3, that when the Federal contribution is relatively large (for example, $300 million or more) and the payment error rate is even moderately higher than the target level of .03 (say .05 or more), the expected proportion of the Federal contribution that is withheld increases with the size of the Federal subsample, assuming that the subsampling rate remains constant. The proportion increases quite rapidly for the smaller sample sizes but at modest rates of increase for sample sizes greater than about 250. The proportion disallowed increases with increasing values of σ_x. Moreover, at any sample size the proportion disallowed is very small if the true payment error rate is less than 5 percent.

For smaller Federal contributions, the proportion no longer increases monotonically with sample size. For high values of the payment error rate, e.g., R=.07, there is a sample size for which the proportion is maximum. However, the curve is quite flat in the neighborhood of the maximum, so that the proportion varies only a little over a broad range of sample sizes. If the payment error rate is low, say below 5 percent, the Federal gain may well be negative, and increasingly negative as sample size increases.

In general, then, from the point of view of maximizing the Federal gain from disallowances after offsetting the costs of sampling, the optimum strategy would be to use quite large samples if the Federal contribution is large and the true payment error rate is relatively high, but to use no sample otherwise. Nevertheless, in the latter case samples are needed to provide assurance that the error is small, in addition to supplying the data needed for feedback information to improve administration.

Table G-1 summarizes, by states, the approximately optimum subsample sizes if the Federal gain from the imposition of disallowances were the

only consideration in determining sample size. The numbers in the table are approximations using data for the last six months of fiscal year 1982, with very rough interpolation of the results summarized in the attached graphs. More accurate computations could be made for each state, but it is doubtful that it would be worth the effort. These results indicate that from this point of view, either no sample would be needed (e.g., if the state's error rate is less than 4 percent or the Federal contribution is quite small), or sample sizes substantially larger than those now used would be desirable. In some cases, no sample at all is called for, because the Federal contribution is so small that the potential return from disallowances cannot pay the cost of a sample. In other cases, no sample is called for because the estimated payment error rate (which was assumed here to be the true rate) was near or below 3 percent. Of course, the "optimum" sample allocation for a particular state could vary widely from year to year; the results in Table G-1 are only illustrative.

We have also estimated the expected gain by simulation using the Test Population A, with 1000 replicate samples for each of three sample sizes. For that population, the true error rate is known, namely .07297. These simulations yielded the results shown in Table G-2. These results are reasonably consistent with the more general results based on the mathematical argument. We note that in Table G-2, the proportion of the Federal contribution that is returned increases with sample size and that the proportion is not highly sensitive to the magnitude of the Federal contribution.

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Table G-1. Rough approximation to optimum size of the Federal subsample if the only consideration were the net return from disallowances

State	Optimum sample size	State	Optimum sample size
Alabama	200	Montana	*
Alaska	*	Nebraska	300
Arizona	170	Nevada	*
Arkansas	*	New Hampshire	*
California	500+	New Jersey	500+
Colorado	300	New Mexico	200
Connecticut	400	New York	500+
Delaware	300	North Carolina	*
District of Columbia	400	North Dakota	*
Florida	350	Ohio	500+
Georgia	350	Oklahoma	*
Hawaii	400	Oregon	300
Idaho	*	Pennsylvania	500+
Illinois	500+	Rhode Island	*
Indiana	*	South Carolina	250
Iowa	*	South Dakota	*
Kansas	*	Tennessee	*
Kentucky	*	Texas	300
Louisiana	350	Utah	*
Maine	*	Vermont	*
Maryland	350	Virginia	300
Massachusetts	500+	Washington	300
Michigan	500+	West Virginia	300
Minnesota	*	Wisconsin	500+
Mississippi	*	Wyoming	*
Missouri	*		

Note: The asterisk (*) denotes that no sample is called for because the Federal contribution is low or the payment error rate is low.

Table G-2. Expected net gain from disallowances, based on simulations from Population A

Federal contribution	n'	Expected gain	Proportion returned
$720,000,000	180	$17,457,000	.024
	80	12,089,000	.017
	50	8,695,000	.012
360,000,000	180	8,621,000	.024
	80	5,998,000	.017
	50	4,319,000	.012
180,000,000	180	4,202,000	.023
	80	2,953,000	.016
	50	2,132,000	.012
90,000,000	180	1,994,000	.022
	80	1,431,000	.016
	50	1,038,000	.012
45,000,000	180	889,100	.020
	80	669,600	.015
	50	491,300	.011

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Figure G-1. Federal gain as proportion of Federal payment share of $20,000,000

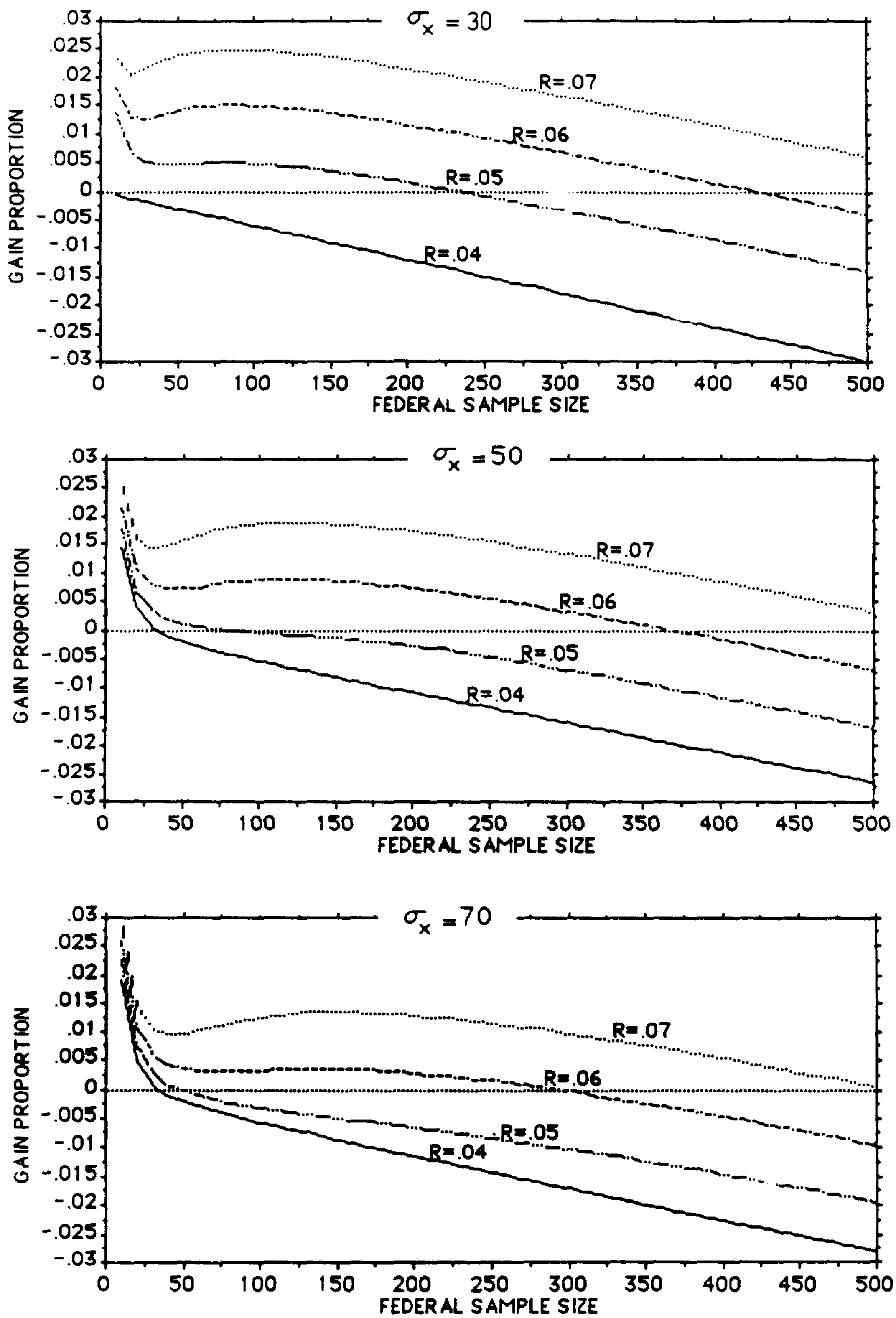

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Figure G-2. Federal gain as proportion of Federal payment share of $50,000,000

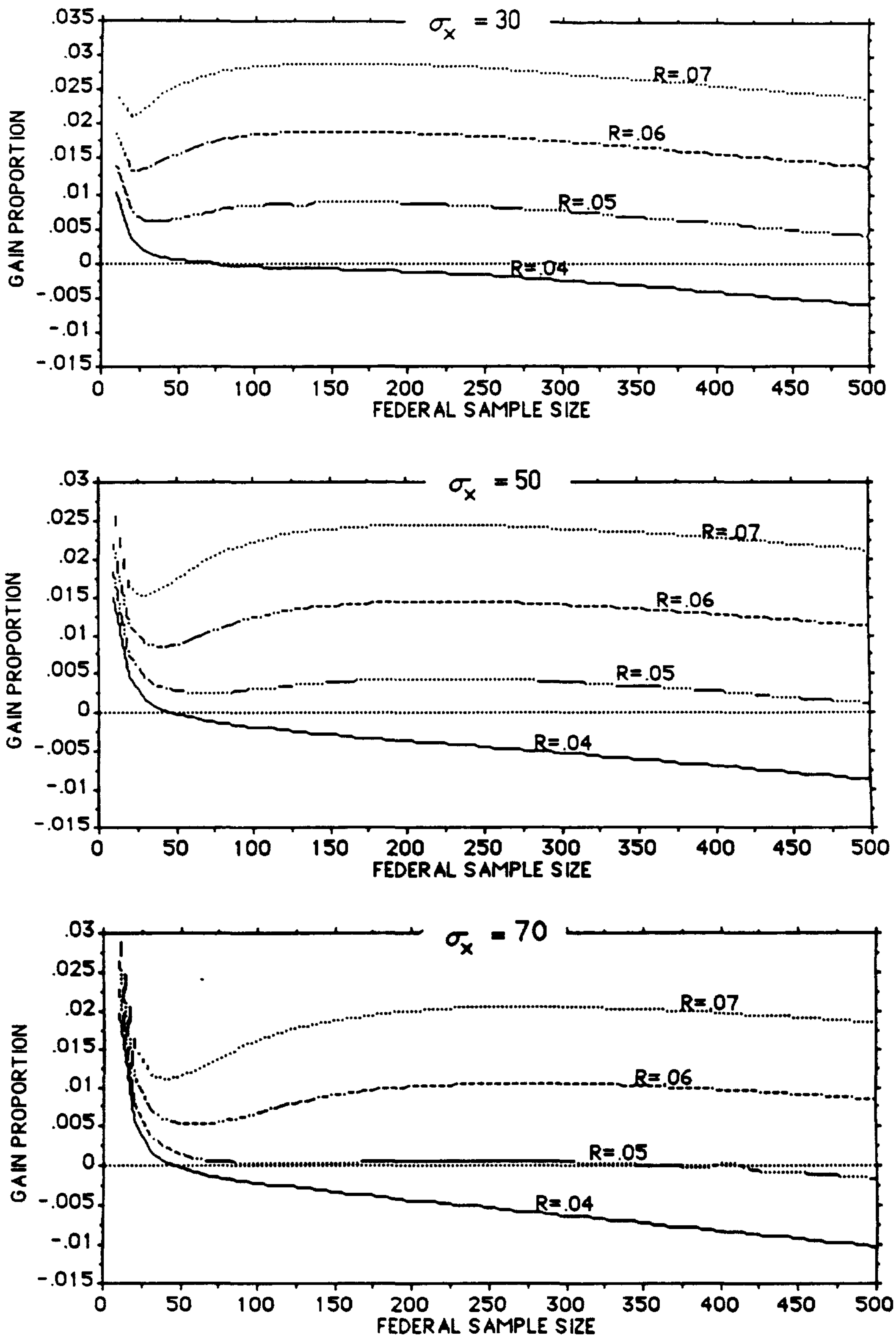

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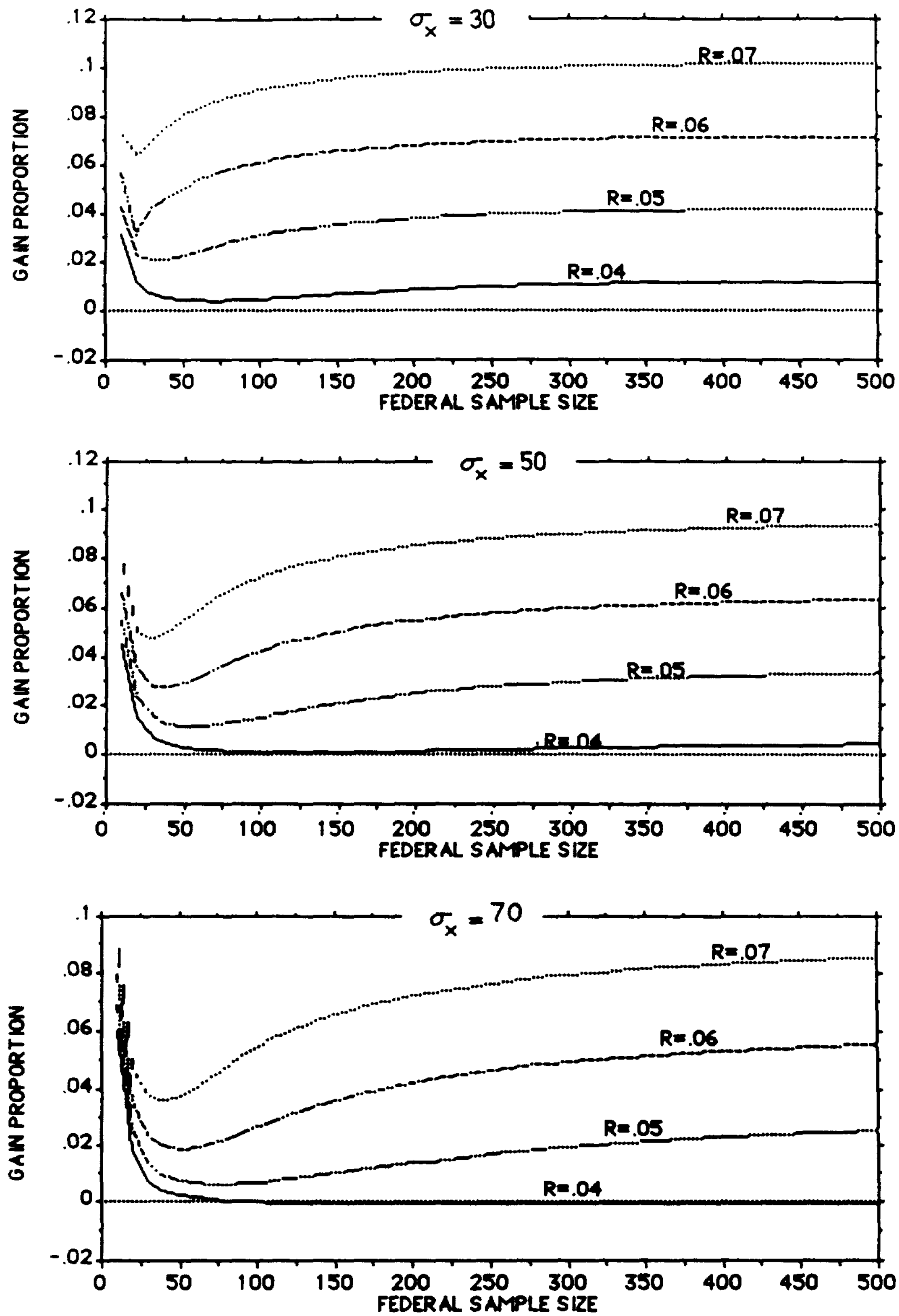
Figure G-3. Federal gain as proportion of Federal payment share of $300,000,000



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APPENDIX H

RULE D FOR COMPUTING DISALLOWANCES BASED ON ACCUMULATIONS ACROSS YEARS

As discussed in Section 3.6, disallowances are computed and assessed annually, and are subject to relatively large sampling errors, even with the larger annual samples in use in the QC program in some states. These large sampling errors can lead to substantially overstated and understated disallowances. The problem of large overestimates of disallowances in some years would be avoided by use of the lower confidence bound instead of the point estimate. However, with present annual sample sizes, this use would result in large losses to the Federal government by consistently and substantially understating the disallowances that would be assessed if the true payment error rates were known.

A related problem with the current rule for the assessment of disallowances is that disallowances are assessed annually and only when the estimated error rate is above the target rate. Thus, because of sampling variation, a state may be assessed a disallowance when in fact the true payment error rate is equal to or below the target rate. Moreover, since negative disallowances are not permitted, such disallowances would not be compensated for over time. Consequently, a state whose true error rate is moderately above the target rate would, on the average, be assessed a larger disallowance than it would be if the true overpayment error rate were known. Also, a state whose error rate is at or below but near the target rate would, on the average, be assessed disallowances.

To eliminate or substantially reduce these problems we describe a procedure, referred to as Rule D, that accumulates the disallowances across years. This procedure has the effect (assuming approximately equal sample sizes each year) of doubling the sample size in two years, tripling it in three, etc., and thus over a few years greatly reduces the impact of sampling errors. A final settlement of the accumulated disallowances based on the point estimates is made at a time when the

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sampling errors are acceptably small. In the intervening years, cash settlements are assessed on the basis of the lower confidence bound of the accumulated disallowances. The Federal government recovers somewhat less in cash prior to the final settlement date but avoids greatly overassessing some states each year. The procedure also substantially eliminates overassessment of states with error rates near the tolerance.

On a relative basis, the accumulated disallowance based on the lower confidence bound approaches over time the full disallowances based on the point estimates. Thus, while there may be a substantial reduction in the first year and a moderate reduction for a few years in the cash withholding by the Federal government, these cash losses may be deemed acceptable in order to avoid greatly overassessing some states in individual years. Indeed, such a procedure might reduce the controversy now taking place with the states over disallowances, and in fact, might result in substantially greater cash collections than can be obtained by assessing annual disallowances based on point estimates (the present procedure), which leads to assessments but not to cash collections except perhaps with long delay

We have developed 16 examples to illustrate the disallowances computed by Rule D under the differing circumstances illustrated by the examples, and to compare them with disallowances as currently computed (Rule A). Each example is based on specific assumptions for the true error rate and other relevant parameters. For each example, we have computed and displayed the amounts of disallowances that would be assessed over a period of 20 years under the present procedure for computing disallowances, and also for Rule D. The results of these computations appear in Tables H-1 through H-16.

While the accumulations are carried out for 20 years in the illustrative examples, the accumulations could be cut off as soon as the estimated coefficient of variation of the accumulated disallowance is sufficiently small, say 10 or 15 percent. A settlement could then be made and the accumulation process could begin again. The estimated coefficient of variation of the total accumulated disallowance each year (based on the point estimates) is shown in the last column of the tables. The cut-off time would be extended more or less indefinitely for states with overpay-

ment error rates near the target (again by cutting off only if the estimated coefficient of variation of the accumulated disallowance is less than 10 percent or 15 percent). Various minor modifications of this general approach could also be considered.

Rule D is defined more exactly and the illustrative tables are explained more fully in what follows.

Let

A_i = Federal contribution to cost in year i;

$\hat{R}_i$ = Estimated overpayment error rate in year i;

s_i = Estimated standard error of $\hat{R}_i$; and

R_{0i} = Target error rate for year i.

Rule D specifies the cumulative disallowance for year i on the basis of the successive point estimates, $\hat{R}_i$, of the annual error rates, namely

$$\mathcal{D}_i = \mathcal{D}_{i-1} + (\hat{R}_i - R_{0i})\, A_i.$$

The cumulative cash transfer for year i is then based on the lower bound of the confidence interval for the cumulative disallowance:

$$\mathcal{C}_i = \begin{cases} \mathcal{D}_i - t\,\hat{\sigma}(\mathcal{D}_i) & \text{if positive,} \\ 0, & \text{otherwise} \end{cases}$$

where we define

$$\hat{\sigma}^2(\mathcal{D}_i) = \hat{\sigma}^2(\mathcal{D}_{i-1}) + A_i^2\, s_i^2.$$

The cumulative book value of the disallowance is the excess of the cumulative disallowance over the cumulative cash transfer, and is given by

$$\mathcal{B}_i = \mathcal{D}_i - \mathcal{C}_i .$$

Note that these formulas also apply to year 1, with the convention that all values are zero for year 0.

The annual cash transfer for year i is then

$$C_i = \mathcal{C}_i - \mathcal{C}_{i-1}$$

and the annual adjustment to the book disallowance is

$$B_i = \mathcal{B}_i - \mathcal{B}_{i-1} .$$

Note that C_i may be a negative number. A negative C_i could be returned to the state in cash or perhaps treated as a credit against future disallowances. The choice is, of course, a policy decision.

The computation given above for the cumulative disallowance is algebraically equivalent to applying the difference between the weighted averages of $\hat{R}_i$ and R_{0i} to the total Federal contribution up to and including the current year. The weights are the proportions that the annual Federal contributions constitute of the total Federal contributions. To show this, we write

$$\begin{aligned}
\mathcal{D}_i &= \mathcal{D}_{i-1} + (\hat{R}_i - R_{0i})\, A_i \\
&= \mathcal{D}_{i-2} + (\hat{R}_i - R_{0i})\, A_{i-1} + (\hat{R}_i - R_{0i})\, A_i \\
&\;\;\vdots \\
&= \sum_{j=1}^{i} (\hat{R}_j - R_{0j})\, A_j \\
&= \left\{ \sum_{j=1}^{i} \frac{A_j}{\Sigma A_j} \hat{R}_j - \sum_{j=1}^{i} \frac{A_j}{\Sigma A_j} R_{0j} \right\} \sum_{j=1}^{i} A_j .
\end{aligned}$$

Since the samples are independent from year to year, it follows that the variance of $\mathcal{D}_i$ may be estimated by

$$\text{var}\,(\mathcal{D}_i) = \sum_{j=1}^{i} A_j^2 \, s_{\hat{R}_j}^2 \, .$$

The coefficient of variation is therefore estimated by

$$\text{cv}\,(\mathcal{D}_i) = \frac{[\text{var}\,(\mathcal{D}_i)]^{1/2}}{\mathcal{D}_i}$$

$$= \frac{1}{\mathcal{D}_i} \left[\sum_{j=1}^{i} A_j^2 \, s_{\hat{R}_j}^2 \right]^{1/2} .$$

Description of Tables

The 16 examples presented in Tables H-1 through H-16 assume various true overpayment error rates and two levels of sampling error. The assumed parameters are shown at the bottom of each table. The examples show a 20-year history of estimated payment error rates. For Examples 1-12, the true payment error rate is assumed to be constant over the years. For Examples 13-16, the true payment error rates vary over the years, as displayed in the column headed "True error rate."

The second and third columns, headed "Error rate" and "sigma," represent the observed estimates of the overpayment error rate and its standard error. They are derived by random selection from the joint distributions of $\hat{R}$ and $s_{\hat{R}}$ defined by the parameters shown for the example. The simulation of the estimated error rate assumed a normal distribution of the estimated error rate, with the specified standard deviation. The latter corresponds approximately to the Federal sample size shown, and is roughly consistent with values observed in the

QC program. The standard error of the estimated payment error rate ("sigma") was simulated by assuming that it was normally distributed with mean equal to the true standard deviation and variance given by the quantity $\sigma^2 (\beta-1)/4n'$, and with β set equal to 40. This gives variances of $s_{\hat{R}}$ that roughly correspond to variances of estimated standard deviations observed for Test Populations A and B. The simulation also involves the assumption that the correlation "rho" between the estimated error rate and its estimated standard error is .7. This also corresponds roughly to the AFDC experience (as seen in Table C-1 in Appendix C).

The column headed "AFDC" shows the disallowance that would be assessed by the present AFDC procedure (except that the negative disallowances shown in this column would be zeros under the present procedure). The two columns headed "Current Disallowance" show the amounts in the current year, added to or subtracted from the cumulative amounts for the previous year, as described above. Thus, the "Cash" column shows the amount that would be withheld (or perhaps disbursed or credited, if negative) in the specified year, and the "Book" column shows the change for the current year in the amount of the credit on the books. Note that the sum of the cash and book amounts is equal to the figures in the AFDC column, except for rounding errors.

The remaining columns show cumulated values. The error rate shown is the average estimated error rate, up to and including the current year.[1] The accumulated standard error ("sigma") is computed on the basis of each year providing an independent sample; i.e., the variance for a given year is computed on the basis of the fact that the annual samples are independent of one another and assuming that the square of the estimated standard error in each year is an unbiased estimate of the variance of the estimated payment error rate. The "Lower bound" for a given year is computed as the estimated error rate minus 1.645 times the estimated standard error for the cumulative (average) error rate, and thus is the lower bound of the nominal 90 percent symmetric confidence interval. Upper

[1] In practice, the procedure described above for computing the cumulative disallowances by Rule D does not involve the computation of this cumulative error rate. We noted above that, implicitly, the effective cumulative error rate is the weighted average of the annual error rates, weighted by the annual Federal payments. However, since the annual Federal payments are assumed to be constant in these illustrations, no weighting is involved.

confidence bounds are computed in a similar manner, although they play no role in Rule D. The cash and book accumulated disallowances are computed as described above. The column "Desired Disallowance" shows the accumulated disallowances that would be assessed under present procedures if the true error rates were known and used to assess the accumulated disallowance. Consequently, no credit is given in years in which the true error rate is less than the target rate.

The tables illustrate how, as the overpayment error rate approaches the target, the estimated coefficient of variation increases, and no cash settlement is involved under Rule D.

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Table H-1. Federal withholding, Rule D, Example 1

Year	Error rate	sigma	AFDC	Current Disallowance Cash	Current Disallowance Book	Cumulated values: Federal contrib.	Error rate	sigma	Lower bound	Upper bound	Disallowance Cash	Disallowance Book	Desired Disall.	Disall Error	cv
0															
1	0.085	0.00634	55	45	10	1,000	0.0850	0.0063	0.0746	0.0955	45	10	50	-5	0.12
2	0.0762	0.00528	46	43	3	2,000	0.0806	0.0041	0.0738	0.0874	88	14	100	-1	0.08
3	0.0839	0.00857	54	48	6	3,000	0.0817	0.0040	0.0752	0.0882	135	20	150	-5	0.08
4	0.0688	0.00451	39	37	1	4,000	0.0785	0.0032	0.0732	0.0837	173	21	200	6	0.07
5	0.0721	0.00337	42	41	1	5,000	0.0772	0.0026	0.0729	0.0815	214	22	250	14	0.06
6	0.0738	0.00443	44	43	1	6,000	0.0766	0.0023	0.0728	0.0804	257	23	300	20	0.05
7	0.0728	0.00458	43	42	1	7,000	0.0761	0.0021	0.0726	0.0795	299	24	350	27	0.05
8	0.0833	0.00779	53	50	3	8,000	0.0770	0.0021	0.0736	0.0804	349	27	400	24	0.04
9	0.0842	0.00614	54	52	2	9,000	0.0778	0.0020	0.0746	0.0810	401	29	450	20	0.04
10	0.072	0.0067	42	40	2	10,000	0.0772	0.0019	0.0741	0.0803	441	31	500	28	0.04
11	0.0765	0.00362	46	46	1	11,000	0.0771	0.0017	0.0743	0.0800	487	32	550	31	0.04
12	0.0727	0.00457	43	42	1	12,000	0.0768	0.0016	0.0741	0.0795	529	33	600	38	0.04
13	0.0893	0.00722	59	57	2	13,000	0.0777	0.0016	0.0751	0.0804	586	35	650	29	0.03
14	0.0865	0.00678	57	55	2	14,000	0.0784	0.0016	0.0758	0.0810	641	36	700	23	0.03
15	0.0831	0.00647	53	52	2	15,000	0.0787	0.0015	0.0762	0.0812	692	38	750	20	0.03
16	0.0798	0.00584	50	49	1	16,000	0.0788	0.0015	0.0763	0.0812	741	39	800	20	0.03
17	0.0877	0.00621	58	56	1	17,000	0.0793	0.0014	0.0769	0.0817	797	40	850	12	0.03
18	0.0765	0.00571	46	45	1	18,000	0.0791	0.0014	0.0768	0.0814	843	41	900	16	0.03
19	0.0787	0.00574	49	48	1	19,000	0.0791	0.0014	0.0769	0.0813	890	43	950	17	0.03
20	0.0778	0.00585	48	47	1	20,000	0.0790	0.0013	0.0769	0.0812	937	44	1000	19	0.03

Parameters:

True payment error rate	0.08
Standard deviation	0.006
Beta	40
rho	0.7
Sample size, n'	360
Annual Federal contribution	1,000

Note: *** indicates that the coefficient of variation is 10 or greater.

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Table H-2. Federal withholding, Rule D, Example 2

Year	Error rate	sigma	AFDC	Current Disallowance Cash	Current Disallowance Book	Cumulated values: Federal contrib.	Error rate	sigma	Lower bound	Upper bound	Disallowance Cash	Disallowance Book	Desired Disall.	Disall Error	cv
0															
1	0.1021	0.02026	72	39	33	1,000	0.1021	0.0203	0.0687	0.1354	39	33	50	-22	0.28
2	0.0714	0.0124	41	36	6	2,000	0.0867	0.0119	0.0672	0.1063	74	39	100	-13	0.21
3	0.089	0.01207	59	54	5	3,000	0.0875	0.0089	0.0729	0.1021	129	44	150	-22	0.15
4	0.0683	0.00685	38	37	1	4,000	0.0827	0.0069	0.0714	0.0940	165	45	200	-11	0.13
5	0.0778	0.01181	48	44	4	5,000	0.0817	0.0060	0.0719	0.0916	209	49	250	-8	0.12
6	0.0594	0.01481	29	24	6	6,000	0.0780	0.0056	0.0688	0.0871	233	55	300	12	0.12
7	0.0936	0.01483	64	58	5	7,000	0.0802	0.0052	0.0716	0.0888	291	60	350	-2	0.10
8	0.0752	0.01392	45	41	4	8,000	0.0796	0.0049	0.0716	0.0876	332	64	400	3	0.10
9	0.0893	0.01669	59	54	6	9,000	0.0807	0.0047	0.0729	0.0884	386	70	450	-6	0.09
10	0.0765	0.00811	46	45	1	10,000	0.0803	0.0043	0.0731	0.0874	431	71	500	-3	0.09
11	0.0581	0.00709	28	27	1	11,000	0.0782	0.0040	0.0717	0.0848	458	72	550	19	0.08
12	0.0722	0.01004	42	40	2	12,000	0.0777	0.0037	0.0716	0.0839	499	74	600	27	0.08
13	0.0748	0.00673	45	44	1	13,000	0.0775	0.0035	0.0718	0.0833	543	75	650	32	0.07
14	0.0818	0.0146	52	48	4	14,000	0.0778	0.0034	0.0722	0.0834	591	79	700	31	0.07
15	0.0856	0.0107	56	54	2	15,000	0.0783	0.0033	0.0730	0.0837	644	81	750	25	0.07
16	0.0697	0.01019	40	38	2	16,000	0.0778	0.0031	0.0727	0.0829	682	82	800	35	0.07
17	0.0857	0.01192	56	53	2	17,000	0.0783	0.0030	0.0733	0.0832	736	85	850	30	0.06
18	0.0771	0.00865	47	46	1	18,000	0.0782	0.0029	0.0734	0.0830	782	86	900	32	0.06
19	0.0644	0.00441	34	34	0	19,000	0.0775	0.0028	0.0729	0.0820	816	86	950	48	0.06
20	0.0746	0.00892	45	43	1	20,000	0.0773	0.0027	0.0730	0.0817	859	87	1000	53	0.06

Parameters:

True payment error rate	0.08
Standard deviation	0.012
Beta	40
rho	0.7
Sample size, n'	120
Annual Federal contribution	1,000

Note: *** indicates that the coefficient of variation is 10 or greater.

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Table H-3. Federal withholding, Rule D, Example 3

Year	Error rate	sigma	AFDC	Current Disallowance		Cumulated values										
				Cash	Book	Federal contrib.	Error rate	sigma	Lower bound	Upper bound	Disallowance Cash	Disallowance Book	Desired Disall.	Disall Error	cv
0															
1	0.0635	0.00734	33	21	12	1,000	0.0635	0.0073	0.0514	0.0755	21	12	30	-3	0.22
2	0.0634	0.00652	33	29	4	2,000	0.0634	0.0049	0.0554	0.0715	51	16	60	-7	0.15
3	0.0697	0.00668	40	36	3	3,000	0.0655	0.0040	0.0590	0.0720	87	20	90	-17	0.11
4	0.0705	0.00713	40	37	3	4,000	0.0668	0.0035	0.0611	0.0725	124	23	120	-27	0.09
5	0.0541	0.00547	24	22	2	5,000	0.0642	0.0030	0.0593	0.0691	147	24	150	-21	0.09
6	0.0658	0.00658	36	34	2	6,000	0.0645	0.0027	0.0600	0.0689	180	27	180	-27	0.08
7	0.0732	0.00775	43	40	3	7,000	0.0657	0.0026	0.0615	0.0700	220	30	210	-40	0.07
8	0.0588	0.00549	29	28	1	8,000	0.0649	0.0024	0.0610	0.0687	248	31	240	-39	0.07
9	0.0634	0.00725	33	31	2	9,000	0.0647	0.0022	0.0610	0.0684	279	33	270	-42	0.06
10	0.0608	0.00545	31	30	1	10,000	0.0643	0.0021	0.0609	0.0678	309	34	300	-43	0.06
11	0.0543	0.00585	24	23	1	11,000	0.0634	0.0020	0.0602	0.0667	332	36	330	-38	0.06
12	0.0603	0.00693	30	29	2	12,000	0.0632	0.0019	0.0600	0.0663	360	38	360	-39	0.06
13	0.0638	0.00534	34	33	1	13,000	0.0632	0.0018	0.0602	0.0662	393	39	390	-42	0.05
14	0.0468	0.00532	17	16	1	14,000	0.0620	0.0017	0.0592	0.0649	409	40	420	-28	0.05
15	0.0572	0.00513	27	26	1	15,000	0.0617	0.0016	0.0590	0.0644	435	40	450	-26	0.05
16	0.0612	0.00655	31	30	1	16,000	0.0617	0.0016	0.0591	0.0643	465	42	480	-27	0.05
17	0.0656	0.00788	36	34	2	17,000	0.0619	0.0016	0.0593	0.0645	499	44	510	-32	0.05
18	0.0673	0.00565	37	36	1	18,000	0.0622	0.0015	0.0597	0.0647	535	45	540	-40	0.05
19	0.0506	0.00428	21	20	1	19,000	0.0616	0.0014	0.0592	0.0640	555	45	570	-38	0.05
20	0.0552	0.00443	25	25	1	20,000	0.0613	0.0014	0.0590	0.0636	580	46	600	-25	0.04

Parameters:

True payment error rate	0.06
Standard deviation	0.006
Beta	40
rho	0.7
Sample size, n'	360
Annual Federal contribution	1,000

Note: "***" indicates that the coefficient of variation is 10 or greater.

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Table H-4. Federal withholding, Rule D, Example 4

Year	Error rate	sigma	AFDC	Current Disallowance Cash	Current Disallowance Book	Cumulated values: Federal contrib.	Error rate	sigma	Lower bound	Upper bound	Disallowance Cash	Disallowance Book	Desired Disall.	Disall Error	cv
0															
1	0.0549	0.00911	25	10	15	1,000	0.0549	0.0091	0.0399	0.0699	10	15	30	5	0.37
2	0.0656	0.01142	36	27	9	2,000	0.0602	0.0073	0.0482	0.0722	36	24	60	0	0.24
3	0.0644	0.01016	34	29	5	3,000	0.0616	0.0059	0.0518	0.0714	66	29	90	-5	0.19
4	0.0608	0.01428	31	23	8	4,000	0.0614	0.0057	0.0520	0.0708	88	38	120	-6	0.18
5	0.0617	0.01097	32	28	4	5,000	0.0615	0.0051	0.0531	0.0698	116	42	150	-7	0.16
6	0.0656	0.01257	36	31	5	6,000	0.0621	0.0047	0.0544	0.0699	146	47	180	-13	0.15
7	0.0617	0.01567	32	25	7	7,000	0.0621	0.0046	0.0545	0.0697	171	53	210	-15	0.14
8	0.0712	0.01524	41	36	6	8,000	0.0632	0.0045	0.0559	0.0706	207	59	240	-26	0.13
9	0.0613	0.01274	31	28	4	9,000	0.0630	0.0042	0.0561	0.0699	235	62	270	-27	0.13
10	0.0675	0.01284	37	34	3	10,000	0.0634	0.0040	0.0569	0.0700	269	66	300	-34	0.12
11	0.0718	0.01374	42	38	4	11,000	0.0642	0.0038	0.0579	0.0705	307	70	330	-48	0.11
12	0.0639	0.01588	34	29	5	12,000	0.0642	0.0038	0.0580	0.0704	336	74	360	-50	0.11
13	0.065	0.01047	35	33	2	13,000	0.0642	0.0036	0.0584	0.0701	369	76	390	-55	0.10
14	0.0672	0.01381	37	34	3	14,000	0.0645	0.0035	0.0588	0.0701	403	80	420	-62	0.10
15	0.0457	0.00558	16	15	1	15,000	0.0632	0.0033	0.0579	0.0686	418	80	450	-48	0.10
16	0.0743	0.01442	44	41	3	16,000	0.0639	0.0032	0.0587	0.0691	459	84	480	-62	0.09
17	0.0813	0.01207	51	49	2	17,000	0.0649	0.0031	0.0599	0.0700	508	86	510	-84	0.09
18	0.0616	0.01232	32	29	2	18,000	0.0647	0.0030	0.0598	0.0696	537	88	540	-85	0.09
19	0.0634	0.01654	33	29	4	19,000	0.0647	0.0030	0.0598	0.0695	566	92	570	-88	0.09
20	0.0661	0.01333	36	34	3	20,000	0.0647	0.0029	0.0600	0.0695	600	95	600	-95	0.08

Parameters:

True payment error rate	0.06
Standard deviation	0.012
Beta	40
rho	0.7
Sample size, n'	120
Annual Federal contribution	1,000

Note: *** indicates that the coefficient of variation is 10 or greater.

Table H-5. Federal withholding, Rule D, Example 5

Year	Error rate	sigma	AFDC	Current Disallowance		Cumulated values									
						Federal contrib.	Error rate	sigma	Lower bound	Upper bound	Disallowance		Desired Disall.	Disall Error	cv
				Cash	Book						Cash	Book			
0															
1	0.0385	0.00543	8	0	8	1,000	0.0385	0.0054	0.0295	0.0474	0	8	10	2	0.64
2	0.0481	0.00707	18	12	6	2,000	0.0433	0.0045	0.0359	0.0506	12	15	20	-7	0.34
3	0.039	0.00523	9	7	2	3,000	0.0418	0.0034	0.0362	0.0475	18	17	30	-5	0.29
4	0.0315	0.00364	1	0	1	4,000	0.0392	0.0027	0.0347	0.0438	19	18	40	3	0.30
5	0.0388	0.00517	9	7	2	5,000	0.0392	0.0024	0.0352	0.0431	26	20	50	4	0.26
6	0.0418	0.00581	12	10	2	6,000	0.0396	0.0022	0.0359	0.0433	36	22	60	2	0.23
7	0.0303	0.00404	0	-1	1	7,000	0.0383	0.0020	0.0350	0.0416	35	23	70	12	0.24
8	0.0401	0.00604	10	8	2	8,000	0.0385	0.0019	0.0354	0.0416	43	25	80	12	0.22
9	0.0369	0.00491	7	6	1	9,000	0.0383	0.0018	0.0354	0.0413	49	26	90	15	0.21
10	0.035	0.00537	5	4	1	10,000	0.0380	0.0017	0.0352	0.0408	52	28	100	20	0.21
11	0.0352	0.00577	5	4	2	11,000	0.0377	0.0016	0.0351	0.0404	56	29	110	25	0.21
12	0.052	0.00822	22	19	3	12,000	0.0389	0.0016	0.0362	0.0416	75	32	120	13	0.18
13	0.0288	0.00542	-1	-2	1	13,000	0.0382	0.0016	0.0356	0.0407	72	34	130	24	0.19
14	0.0368	0.0043	7	6	1	14,000	0.0381	0.0015	0.0356	0.0405	78	34	140	27	0.18
15	0.0392	0.00584	9	8	1	15,000	0.0381	0.0014	0.0358	0.0405	86	36	150	28	0.18
16	0.0477	0.00635	18	16	2	16,000	0.0387	0.0014	0.0364	0.0410	102	37	160	20	0.16
17	0.0418	0.00637	12	10	1	17,000	0.0389	0.0014	0.0366	0.0412	113	39	170	18	0.15
18	0.0442	0.00644	14	13	1	18,000	0.0392	0.0014	0.0370	0.0414	126	40	180	14	0.15
19	0.0399	0.0063	10	9	1	19,000	0.0392	0.0013	0.0371	0.0414	134	41	190	14	0.14
20	0.0463	0.00686	16	15	2	20,000	0.0396	0.0013	0.0374	0.0417	149	43	200	8	0.14

Parameters:

True payment error rate	0.04
Standard deviation	0.006
Beta	40
rho	0.7
Sample size, n'	360
Annual Federal contribution	1,000

Note: *** indicates that the coefficient of variation is 10 or greater.

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Table H-6. Federal withholding, Rule D, Example 6

Year	Error rate	sigma	AFDC	Current Disallowance Cash	Current Disallowance Book	Cumulated values: Federal contrib.	Error rate	sigma	Lower bound	Upper bound	Disallowance Cash	Disallowance Book	Desired Disall.	Disall Error	CV
0															
1	0.0433	0.01036	13	0	13	1,000	0.0433	0.0104	0.0263	0.0604	0	13	10	-3	0.78
2	0.0405	0.00997	11	0	10	2,000	0.0419	0.0072	0.0301	0.0537	0	24	20	-4	0.60
3	0.0287	0.00517	-1	0	-1	3,000	0.0375	0.0051	0.0291	0.0459	0	23	30	7	0.68
4	0.0738	0.0183	44	27	17	4,000	0.0466	0.0060	0.0368	0.0564	27	39	40	-26	0.36
5	0.0306	0.00772	1	-1	2	5,000	0.0434	0.0050	0.0351	0.0516	26	41	50	-17	0.37
6	0.0247	0.00757	-5	-7	2	6,000	0.0403	0.0044	0.0331	0.0475	19	43	60	-2	0.42
7	0.0483	0.01305	18	13	5	7,000	0.0414	0.0042	0.0346	0.0483	32	48	70	-10	0.37
8	0.0503	0.01138	20	17	4	8,000	0.0425	0.0039	0.0361	0.0490	49	52	80	-21	0.31
9	0.0366	0.00908	7	5	2	9,000	0.0419	0.0036	0.0359	0.0479	53	54	90	-17	0.31
10	0.0391	0.01332	9	5	4	10,000	0.0416	0.0035	0.0358	0.0474	58	58	100	-16	0.30
11	0.0543	0.01586	24	19	6	11,000	0.0428	0.0035	0.0370	0.0485	77	64	110	-31	0.28
12	0.033	0.01354	3	-1	4	12,000	0.0419	0.0034	0.0363	0.0476	76	67	120	-23	0.29
13	0.0127	0.00849	-17	-19	1	13,000	0.0397	0.0032	0.0344	0.0450	57	69	130	4	0.33
14	0.0367	0.01018	7	5	2	14,000	0.0395	0.0031	0.0344	0.0445	62	71	140	7	0.32
15	0.0581	0.01267	28	25	3	15,000	0.0407	0.0030	0.0358	0.0456	87	74	150	-11	0.28
16	0.0293	0.00336	-1	-1	0	16,000	0.0400	0.0028	0.0354	0.0446	86	74	160	0	0.28
17	0.0642	0.02001	34	27	7	17,000	0.0414	0.0029	0.0367	0.0462	113	81	170	-24	0.25
18	0.0461	0.01662	16	12	4	18,000	0.0417	0.0029	0.0369	0.0464	125	86	180	-31	0.25
19	0.0214	0.00754	-9	-10	1	19,000	0.0406	0.0028	0.0361	0.0452	115	86	190	-12	0.26
20	0.0225	0.0063	-7	-8	1	20,000	0.0397	0.0026	0.0354	0.0441	107	87	200	6	0.27

Parameters:

True payment error rate	0.04
Standard deviation	0.012
Beta	40
rho	0.7
Sample size, n'	120
Annual Federal contribution	1,000

Note: *** indicates that the coefficient of variation is 10 or greater.

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Table H-7. Federal withholding, Rule D, Example 7

Year	Error rate	sigma	AFDC	Current Disallowance		Cumulated values									
						Federal contrib.	Error rate	sigma	Lower bound	Upper bound	Disallowance		Desired Disall.	Disall Error	CV
				Cash	Book						Cash	Book			
0															
1	0.0288	0.0057	-1	0	-1	1,000	0.0288	0.0057	0.0194	0.0381	0	-1	3	4	4.65
2	0.0306	0.00548	1	0	1	2,000	0.0297	0.0040	0.0232	0.0362	0	-1	6	7	***
3	0.0314	0.00553	1	0	1	3,000	0.0303	0.0032	0.0250	0.0356	0	1	9	8	***
4	0.0269	0.00464	-3	0	-3	4,000	0.0294	0.0027	0.0250	0.0338	0	-2	12	14	4.62
5	0.025	0.00535	-5	0	-5	5,000	0.0285	0.0024	0.0246	0.0325	0	-7	15	22	1.63
6	0.0357	0.00609	6	0	6	6,000	0.0297	0.0022	0.0260	0.0334	0	-2	18	20	7.96
7	0.0421	0.00684	12	0	12	7,000	0.0315	0.0022	0.0279	0.0350	0	10	21	11	1.45
8	0.0297	0.00469	0	0	0	8,000	0.0313	0.0020	0.0280	0.0345	0	10	24	14	1.56
9	0.0288	0.00559	-1	0	-1	9,000	0.0310	0.0019	0.0279	0.0341	0	9	27	18	1.87
10	0.0381	0.00621	8	0	8	10,000	0.0317	0.0018	0.0288	0.0346	0	17	30	13	1.05
11	0.0269	0.00646	-3	0	-3	11,000	0.0313	0.0017	0.0284	0.0341	0	14	33	19	1.36
12	0.0375	0.00649	8	0	8	12,000	0.0318	0.0017	0.0290	0.0345	0	21	36	15	0.94
13	0.0272	0.00513	-3	0	-3	13,000	0.0314	0.0016	0.0288	0.0341	0	19	39	20	1.11
14	0.0375	0.00707	8	0	8	14,000	0.0319	0.0016	0.0293	0.0344	0	26	42	18	0.84
15	0.0348	0.00578	5	0	5	15,000	0.0321	0.0015	0.0296	0.0345	0	31	45	14	0.73
16	0.0173	0.0047	-13	0	-13	16,000	0.0311	0.0014	0.0288	0.0335	0	18	48	30	1.27
17	0.0384	0.00702	8	0	8	17,000	0.0316	0.0014	0.0292	0.0339	0	27	51	24	0.90
18	0.0207	0.0059	-9	0	-9	18,000	0.0310	0.0014	0.0287	0.0332	0	17	54	37	1.43
19	0.0312	0.00559	1	0	1	19,000	0.0310	0.0013	0.0288	0.0332	0	19	57	30	1.37
20	0.0429	0.0067	13	0	13	20,000	0.0316	0.0013	0.0294	0.0337	0	32	60	28	0.84

Note: *** indicates that the coefficient of variation is 10 or greater.

Parameters:

True payment error rate	0.033
Standard deviation	0.006
Beta	40
rho	0.7
Sample size, n'	360
Annual Federal contribution	1,000

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Table H-8. Federal withholding, Rule D, Example 8

Year	Error rate	sigma	AFDC	Current Disallowance Cash	Current Disallowance Book	Cumulated values: Federal contrib.	Error rate	sigma	Lower bound	Upper bound	Disallowance Cash	Disallowance Book	Desired Disall.	Disall Error	CV
0															
1	0.0249	0.01007	-5	0	-5	1,000	0.0249	0.0101	0.0083	0.0415	0	-5	3	8	1.98
2	0.0464	0.01336	16	0	16	2,000	0.0357	0.0084	0.0219	0.0494	0	11	6	-5	1.47
3	0.0277	0.01395	-2	0	-2	3,000	0.0330	0.0073	0.0211	0.0450	0	9	9	0	2.41
4	0.0237	0.0124	-6	0	-6	4,000	0.0307	0.0063	0.0204	0.0410	0	3	12	9	9.23
5	0.0241	0.01433	-6	0	-6	5,000	0.0294	0.0058	0.0199	0.0389	0	-3	15	18	9.16
6	0.0301	0.0067	0	0	0	6,000	0.0295	0.0049	0.0214	0.0376	0	-3	18	21	9.69
7	0.0245	0.00938	-6	0	-6	7,000	0.0288	0.0044	0.0215	0.0361	0	-9	21	30	3.63
8	0.0238	0.00855	-6	0	-6	8,000	0.0282	0.0040	0.0215	0.0348	0	-15	24	39	2.19
9	0.0536	0.01358	24	0	24	9,000	0.0310	0.0039	0.0246	0.0374	0	9	27	18	3.95
10	0.0338	0.01014	4	0	4	10,000	0.0313	0.0036	0.0253	0.0373	0	13	30	17	2.87
11	0.0314	0.01429	1	0	1	11,000	0.0313	0.0036	0.0254	0.0371	0	14	33	19	2.78
12	0.0293	0.01296	-1	0	-1	12,000	0.0311	0.0034	0.0255	0.0368	0	13	36	23	3.08
13	0.029	0.01213	-1	0	-1	13,000	0.0310	0.0033	0.0255	0.0364	0	12	39	27	3.45
14	0.0663	0.01572	36	0	36	14,000	0.0335	0.0033	0.0281	0.0389	0	49	42	-7	0.94
15	0.0301	0.01491	0	0	0	15,000	0.0333	0.0032	0.0280	0.0385	0	49	45	-4	0.99
16	0.0267	0.00848	-3	0	-3	16,000	0.0328	0.0031	0.0278	0.0379	0	46	48	2	1.07
17	0.0326	0.01182	3	0	3	17,000	0.0328	0.0030	0.0280	0.0377	0	48	51	3	1.05
18	0.0568	0.01634	27	0	27	18,000	0.0342	0.0029	0.0293	0.0390	0	75	54	-21	0.71
19	0.0371	0.01461	7	0	7	19,000	0.0343	0.0029	0.0296	0.0391	0	82	57	-25	0.67
20	0.0156	0.00795	-14	0	-14	20,000	0.0334	0.0028	0.0288	0.0379	0	68	60	-8	0.82

Parameters:

True payment error rate	0.033
Standard deviation	0.012
Beta	40
rho	0.7
Sample size, n'	120
Annual Federal contribution	1,000

Note: *** indicates that the coefficient of variation is 10 or greater.

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Table H-9. Federal withholding, Rule D, Example 9

Year	Error rate	sigma	AFDC	Current Disallowance		Cumulated values									
				Cash	Book	Federal contrib.	Error rate	sigma	Lower bound	Upper bound	Disallowance Cash	Disallowance Book	Desired Disall.	Disall Error	cv
0															
1	0.0341	0.00672	4	0	4	1,000	0.0341	0.0067	0.0230	0.0451	0	4	0	-4	1.65
2	0.025	0.00597	-5	0	-5	2,000	0.0296	0.0045	0.0222	0.0369	0	-1	0	1	***
3	0.0349	0.00662	5	0	5	3,000	0.0313	0.0037	0.0252	0.0375	0	4	0	-4	2.77
4	0.0303	0.00736	0	0	0	4,000	0.0311	0.0033	0.0256	0.0366	0	4	0	-4	3.07
5	0.0275	0.0075	-3	0	-3	5,000	0.0304	0.0031	0.0253	0.0354	0	2	0	-2	8.31
6	0.0327	0.00574	3	0	3	6,000	0.0308	0.0027	0.0263	0.0352	0	5	0	-5	3.62
7	0.0229	0.00611	-7	0	-7	7,000	0.0296	0.0025	0.0255	0.0337	0	-3	0	3	6.90
8	0.0411	0.00669	11	0	11	8,000	0.0311	0.0023	0.0272	0.0349	0	9	0	-9	2.18
9	0.0296	0.00666	0	0	0	9,000	0.0309	0.0022	0.0273	0.0345	0	8	0	-8	2.44
10	0.0267	0.005	-3	0	-3	10,000	0.0305	0.0020	0.0271	0.0339	0	5	0	-5	4.23
11	0.0266	0.00561	-3	0	-3	11,000	0.0301	0.0019	0.0270	0.0333	0	1	0	-1	***
12	0.0193	0.00442	-11	0	-11	12,000	0.0292	0.0018	0.0263	0.0322	0	-9	0	9	2.34
13	0.0318	0.00652	2	0	2	13,000	0.0294	0.0017	0.0266	0.0323	0	-7	0	7	3.05
14	0.0328	0.00647	3	0	3	14,000	0.0297	0.0017	0.0269	0.0324	0	-5	0	5	5.14
15	0.031	0.00495	1	0	1	15,000	0.0298	0.0016	0.0271	0.0324	0	-4	0	4	6.69
16	0.0341	0.00732	4	0	4	16,000	0.0300	0.0016	0.0274	0.0326	0	1	0	-1	***
17	0.027	0.00615	-3	0	-3	17,000	0.0299	0.0015	0.0273	0.0324	0	-2	0	2	***
18	0.0201	0.00516	-10	0	-10	18,000	0.0293	0.0015	0.0269	0.0317	0	-12	0	12	2.13
19	0.0397	0.00699	10	0	10	19,000	0.0299	0.0014	0.0275	0.0322	0	-3	0	3	***
20	0.0319	0.00664	2	0	2	20,000	0.0300	0.0014	0.0276	0.0323	0	-1	0	1	***

Parameters:

True payment error rate	0.03
Standard deviation	0.006
Beta	40
rho	0.7
Sample size, n'	360
Annual Federal contribution	1,000

Note: *** indicates that the coefficient of variation is 10 or greater.

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Table H-10. Federal withholding, Rule D, Example 10

Year	Error rate	sigma	AFDC	Current Disallowance: Cash	Current Disallowance: Book	Cumulated values: Federal contrib.	Error rate	sigma	Lower bound	Upper bound	Disallowance: Cash	Disallowance: Book	Desired Disall.	Disall Error	cv
0															
1	0.0297	0.01107	0	0	0	1,000	0.0297	0.0111	0.0115	0.0479	0	0	0	0	***
2	0.0153	0.01068	-15	0	-15	2,000	0.0225	0.0077	0.0098	0.0351	0	-15	0	15	1.02
3	0.0443	0.01591	14	0	14	3,000	0.0298	0.0074	0.0176	0.0419	0	-1	0	1	***
4	0.0567	0.01551	27	0	27	4,000	0.0365	0.0068	0.0254	0.0476	0	26	0	-26	1.04
5	0.0279	0.01031	-2	0	-2	5,000	0.0348	0.0058	0.0253	0.0443	0	24	0	-24	1.21
6	0.0348	0.01451	5	0	5	6,000	0.0348	0.0054	0.0259	0.0437	0	29	0	-29	1.13
7	0.009	0.00564	-21	0	-21	7,000	0.0311	0.0047	0.0234	0.0388	0	8	0	-8	4.24
8	0.0653	0.01938	35	0	35	8,000	0.0354	0.0048	0.0275	0.0432	0	43	0	-43	0.89
9	0.0411	0.01314	11	0	11	9,000	0.0360	0.0045	0.0286	0.0434	0	54	0	-54	0.74
10	0.0178	0.01002	-12	0	-12	10,000	0.0342	0.0042	0.0274	0.0410	0	42	0	-42	0.99
11	0.0187	0.012	-11	0	-11	11,000	0.0328	0.0039	0.0263	0.0393	0	31	0	-31	1.41
12	0.042	0.01023	12	0	12	12,000	0.0336	0.0037	0.0275	0.0396	0	43	0	-43	1.04
13	0.0428	0.01491	13	0	13	13,000	0.0343	0.0036	0.0283	0.0402	0	55	0	-55	0.85
14	0.0195	0.00788	-10	0	-10	14,000	0.0332	0.0034	0.0276	0.0388	0	45	0	-45	1.06
15	0.0233	0.0079	-7	0	-7	15,000	0.0325	0.0032	0.0273	0.0378	0	38	0	-38	1.26
16	0.0214	0.00805	-9	0	-9	16,000	0.0319	0.0031	0.0268	0.0369	0	30	0	-30	1.65
17	0.0326	0.01118	3	0	3	17,000	0.0319	0.0029	0.0270	0.0367	0	32	0	-32	1.55
18	0.0396	0.0133	10	0	10	18,000	0.0323	0.0029	0.0276	0.0371	0	42	0	-42	1.24
19	0.0128	0.00783	-17	0	-17	19,000	0.0313	0.0028	0.0268	0.0358	0	25	0	-25	2.12
20	0.0289	0.01357	-1	0	-1	20,000	0.0312	0.0027	0.0267	0.0356	0	24	0	-24	2.29

Parameters:

True payment error rate	0.03
Standard deviation	0.012
Beta	40
rho	0.7
Sample size, n'	120
Annual Federal contribution	1,000

Note: *** indicates that the coefficient of variation is 10 or greater.

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Table H-11. Federal withholding, Rule D, Example 11

Year	Error rate	sigma	AFDC	Current Disallowance Cash	Current Disallowance Book	Cumulated values: Federal contrib.	Error rate	sigma	Lower bound	Upper bound	Disallowance Cash	Disallowance Book	Desired Disall.	Disall Error	cv
0															
1	0.022	0.00672	-8	0	-8	1,000	0.0220	0.0067	0.0110	0.0331	0	-8	-5	3	0.84
2	0.0208	0.00642	-9	-2	-7	2,000	0.0214	0.0046	0.0138	0.0291	-2	-15	-10	7	0.54
3	0.0267	0.00656	-3	0	-3	3,000	0.0232	0.0038	0.0169	0.0294	-2	-19	-15	5	0.56
4	0.0173	0.0068	-13	-10	-3	4,000	0.0217	0.0033	0.0163	0.0272	-11	-22	-20	13	0.40
5	0.0327	0.00773	3	6	-3	5,000	0.0239	0.0031	0.0189	0.0290	-5	-25	-25	5	0.50
6	0.0188	0.00504	-11	-10	-1	6,000	0.0231	0.0027	0.0186	0.0275	-15	-27	-30	12	0.39
7	0.0226	0.00652	-7	-5	-2	7,000	0.0230	0.0025	0.0189	0.0271	-20	-29	-35	14	0.36
8	0.0234	0.0068	-7	-4	-2	8,000	0.0231	0.0023	0.0192	0.0269	-25	-31	-40	18	0.34
9	0.0128	0.00394	-17	-16	-1	9,000	0.0219	0.0021	0.0184	0.0254	-41	-31	-45	28	0.26
10	0.0256	0.00678	-4	-2	-2	10,000	0.0223	0.0020	0.0189	0.0256	-44	-33	-50	27	0.26
11	0.0245	0.00584	-6	-4	-1	11,000	0.0225	0.0019	0.0193	0.0256	-48	-35	-55	28	0.26
12	0.0263	0.00576	-4	-2	-1	12,000	0.0228	0.0018	0.0198	0.0258	-50	-36	-60	28	0.25
13	0.0372	0.00725	7	9	-2	13,000	0.0239	0.0018	0.0210	0.0268	-41	-38	-65	14	0.29
14	0.0249	0.00588	-5	-4	-1	14,000	0.0240	0.0017	0.0212	0.0268	-45	-39	-70	14	0.28
15	0.0236	0.00425	-6	-6	-1	15,000	0.0240	0.0016	0.0213	0.0266	-51	-40	-75	18	0.27
16	0.0281	0.00633	-2	-1	-1	16,000	0.0242	0.0016	0.0216	0.0268	-51	-41	-80	19	0.27
17	0.0191	0.00486	-11	-10	-1	17,000	0.0239	0.0015	0.0215	0.0264	-62	-42	-85	18	0.25
18	0.0235	0.0067	-7	-5	-1	18,000	0.0239	0.0015	0.0215	0.0263	-67	-43	-90	20	0.24
19	0.0207	0.0052	-9	-8	-1	19,000	0.0237	0.0014	0.0214	0.0260	-75	-44	-95	24	0.22
20	0.0215	0.00576	-8	-7	-1	20,000	0.0236	0.0014	0.0214	0.0259	-83	-45	-100	28	0.21

Parameters:

True payment error rate	0.025
Standard deviation	0.006
Beta	40
rho	0.7
Sample size, n'	360
Annual Federal contribution	1,000

Note: *** indicates that the coefficient of variation is 10 or greater.

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Table H-12. Federal withholding, Rule D, Example 12

Year	Error rate	sigma	AFDC	Current Disallowance: Cash	Current Disallowance: Book	Cumulated values: Federal contrib.	Error rate	sigma	Lower bound	Upper bound	Disallowance: Cash	Disallowance: Book	Desired Disall.	Disall Error	cv
0															
1	0.03	0.01589	0	0	0	1,000	0.0300	0.0159	0.0039	0.0562	0	0	-5	-5	***
2	0.0321	0.01311	2	0	2	2,000	0.0311	0.0103	0.0142	0.0480	0	2	-10	-12	9.41
3	0.0163	0.00753	-14	0	-14	3,000	0.0262	0.0073	0.0141	0.0382	0	-12	-15	-3	1.91
4	0.0343	0.01238	4	0	4	4,000	0.0282	0.0063	0.0178	0.0386	0	-7	-20	-13	3.50
5	0.0186	0.00956	-11	0	-11	5,000	0.0263	0.0054	0.0174	0.0351	0	-19	-25	-6	1.45
6	0.0145	0.01014	-15	0	-15	6,000	0.0243	0.0048	0.0164	0.0322	0	-34	-30	4	0.85
7	0.0423	0.01276	12	0	12	7,000	0.0269	0.0045	0.0195	0.0343	0	-22	-35	-13	1.45
8	0.0167	0.00724	-13	0	-13	8,000	0.0256	0.0040	0.0190	0.0323	0	-35	-40	-5	0.92
9	0.0282	0.00842	-2	0	-2	9,000	0.0259	0.0037	0.0198	0.0320	0	-37	-45	-8	0.91
10	0.0408	0.01481	11	0	11	10,000	0.0274	0.0037	0.0214	0.0334	0	-26	-50	-24	1.40
11	0.0318	0.01293	2	0	2	11,000	0.0278	0.0035	0.0220	0.0336	0	-24	-55	-31	1.59
12	0.0222	0.00971	-8	0	-8	12,000	0.0273	0.0033	0.0218	0.0328	0	-32	-60	-28	1.24
13	0.0185	0.01127	-12	0	-12	13,000	0.0266	0.0032	0.0214	0.0319	0	-44	-65	-21	0.95
14	0.0326	0.01342	3	0	3	14,000	0.0271	0.0031	0.0219	0.0322	0	-41	-70	-29	1.06
15	0.0207	0.01003	-9	0	-9	15,000	0.0266	0.0030	0.0217	0.0315	0	-50	-75	-25	0.89
16	0.04	0.01304	10	0	10	16,000	0.0275	0.0029	0.0227	0.0323	0	-40	-80	-40	1.15
17	0.0496	0.01913	20	0	20	17,000	0.0288	0.0030	0.0239	0.0337	0	-21	-85	-64	2.42
18	0.0087	0.0073	-21	0	-21	18,000	0.0277	0.0028	0.0230	0.0323	0	-42	-90	-48	1.21
19	0.0474	0.01625	17	0	17	19,000	0.0287	0.0028	0.0241	0.0333	0	-25	-95	-70	2.17
20	0.0306	0.01507	1	0	1	20,000	0.0288	0.0028	0.0242	0.0334	0	-24	-100	-76	2.31

Parameters:

True payment error rate	0.025
Standard deviation	0.012
Beta	40
rho	0.7
Sample size, n'	120
Annual Federal contribution	1,000

Note: *** indicates that the coefficient of variation is 10 or greater.

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Table H-13. Federal withholding, Rule D, Example 13

Year	Error rate	sigma	AFDC	Current Disallowance Cash	Current Disallowance Book	Cumulated values: Federal contrib.	Error rate	sigma	Lower bound	Upper bound	Disallowance Cash	Disallowance Book	Desired Disall.	True error rate	Disall. Error	cv
0																
1	0.075	0.00634	45	35	10	1,000	0.0750	0.0063	0.0646	0.0855	35	10	40	0.070	-5	0.1409
2	0.0612	0.00528	31	28	3	2,000	0.0681	0.0041	0.0613	0.0749	63	14	75	0.065	-1	0.1083
3	0.0639	0.00857	34	28	6	3,000	0.0667	0.0040	0.0602	0.0732	90	20	105	0.060	-5	0.1081
4	0.0488	0.00451	19	17	1	4,000	0.0622	0.0032	0.0570	0.0675	108	21	135	0.060	6	0.0987
5	0.0421	0.00337	12	11	1	5,000	0.0582	0.0026	0.0539	0.0625	119	22	155	0.050	14	0.0934
6	0.0438	0.00443	14	13	1	6,000	0.0558	0.0023	0.0520	0.0596	132	23	175	0.050	20	0.0897
7	0.0428	0.00458	13	12	1	7,000	0.0539	0.0021	0.0505	0.0574	144	24	195	0.050	27	0.0873
8	0.0533	0.00779	23	20	3	8,000	0.0539	0.0021	0.0505	0.0573	164	27	215	0.050	24	0.0868
9	0.0492	0.00614	19	17	2	9,000	0.0533	0.0020	0.0501	0.0566	181	29	230	0.045	20	0.0841
10	0.037	0.0067	7	5	2	10,000	0.0517	0.0019	0.0486	0.0548	186	31	245	0.045	28	0.087
11	0.0365	0.00362	6	6	1	11,000	0.0503	0.0017	0.0475	0.0532	192	32	255	0.040	31	0.0861
12	0.0327	0.00457	3	2	1	12,000	0.0489	0.0016	0.0461	0.0516	194	33	265	0.040	39	0.0874
13	0.0493	0.00722	19	17	2	13,000	0.0489	0.0016	0.0462	0.0516	211	35	275	0.040	29	0.0857
14	0.0415	0.00678	12	10	2	14,000	0.0484	0.0016	0.0458	0.0510	221	36	280	0.035	23	0.086
15	0.0381	0.00647	8	7	2	15,000	0.0477	0.0015	0.0452	0.0502	227	38	285	0.035	20	0.0869
16	0.0298	0.00584	0	-1	1	16,000	0.0466	0.0015	0.0441	0.0490	226	39	285	0.030	20	0.0897
17	0.0377	0.00621	8	6	1	17,000	0.0460	0.0014	0.0437	0.0484	232	40	285	0.030	12	0.0901
18	0.0265	0.00571	-4	-5	1	18,000	0.0450	0.0014	0.0426	0.0473	228	41	285	0.030	16	0.0937
19	0.0237	0.00574	-6	-7	1	19,000	0.0438	0.0014	0.0416	0.0461	220	43	280	0.025	17	0.0984
20	0.0228	0.00585	-7	-8	1	20,000	0.0428	0.0013	0.0406	0.0450	212	44	275	0.025	19	0.1037

Parameters:

Varying payment error rate	
Standard deviation	0.006
Beta	40
rho	0.7
Sample size, n'	360
Annual Federal contribution	1,000

Note: *** indicates that the coefficient of variation is 10 or greater.

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Table H-14. Federal withholding, Rule D, Example 14

Year	Error rate	sigma	AFDC	Current Disallowance: Cash	Current Disallowance: Book	Cumulated values: Federal contrib.	Error rate	sigma	Lower bound	Upper bound	Disallowance: Cash	Disallowance: Book	Desired Disall.	True error rate	Disall. Error	cv
0																
1	0.0921	0.02026	62	29	33	1,000	0.0921	0.0203	0.0587	0.1254	29	33	40	0.070	-22	0.33
2	0.0564	0.0124	26	21	6	2,000	0.0742	0.0119	0.0547	0.0938	49	39	75	0.065	-13	0.27
3	0.069	0.01207	39	34	5	3,000	0.0725	0.0089	0.0579	0.0871	84	44	105	0.060	-22	0.21
4	0.0483	0.00685	18	17	1	4,000	0.0664	0.0069	0.0551	0.0777	100	45	135	0.060	-11	0.19
5	0.0478	0.01181	18	14	4	5,000	0.0627	0.0060	0.0529	0.0726	114	49	155	0.050	-9	0.18
6	0.0294	0.01481	-1	-6	6	6,000	0.0572	0.0056	0.0480	0.0663	108	55	175	0.050	12	0.21
7	0.0636	0.01483	34	28	5	7,000	0.0581	0.0052	0.0495	0.0667	136	60	195	0.050	-2	0.19
8	0.0452	0.01392	15	11	4	8,000	0.0565	0.0049	0.0484	0.0645	147	64	215	0.050	3	0.18
9	0.0543	0.01669	24	19	6	9,000	0.0562	0.0047	0.0485	0.0640	166	70	230	0.045	-6	0.18
10	0.0415	0.00811	11	10	1	10,000	0.0548	0.0043	0.0476	0.0619	176	71	245	0.045	-3	0.17
11	0.0181	0.00709	-12	-13	1	11,000	0.0514	0.0040	0.0449	0.0580	163	72	255	0.040	19	0.19
12	0.0322	0.01004	2	0	2	12,000	0.0498	0.0037	0.0437	0.0560	164	74	265	0.040	27	0.19
13	0.0348	0.00673	5	4	1	13,000	0.0487	0.0035	0.0429	0.0544	168	75	275	0.040	32	0.19
14	0.0368	0.0146	7	3	4	14,000	0.0478	0.0034	0.0422	0.0534	171	79	280	0.035	31	0.19
15	0.0406	0.0107	11	9	2	15,000	0.0473	0.0033	0.0420	0.0527	179	81	285	0.035	25	0.19
16	0.0197	0.01019	-10	-12	2	16,000	0.0456	0.0031	0.0405	0.0508	167	82	285	0.030	35	0.20
17	0.0357	0.01192	6	3	2	17,000	0.0450	0.0030	0.0401	0.0500	171	85	285	0.030	30	0.20
18	0.0271	0.00865	-3	-4	1	18,000	0.0440	0.0029	0.0393	0.0488	167	86	285	0.030	32	0.21
19	0.0094	0.00441	-21	-21	0	19,000	0.0422	0.0028	0.0377	0.0467	146	86	280	0.025	48	0.23
20	0.0196	0.00892	-10	-12	1	20,000	0.0411	0.0027	0.0367	0.0454	134	87	275	0.025	53	0.24

Parameters:

Varying payment error rate	
Standard deviation	0.012
Beta	40
rho	0.7
Sample size, n'	120
Annual Federal contribution	1,000

Note: *** indicates that the coefficient of variation is 10 or greater.

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Table H-15. Federal withholding, Rule D, Example 15

Year	Error rate	sigma	AFDC	Current Disallowance Cash	Current Disallowance Book	Cumulated values: Federal contrib.	Error rate	sigma	Lower bound	Upper bound	Disallowance Cash	Disallowance Book	Desired Disall.	True error rate	Disall Error	cv
0																
1	0.055	0.00634	25	15	10	1,000	0.0550	0.0063	0.0446	0.0655	15	10	40	0.070	15	0.25
2	0.0412	0.00528	11	8	3	2,000	0.0481	0.0041	0.0413	0.0549	23	14	75	0.065	39	0.23
3	0.0489	0.00857	19	13	6	3,000	0.0484	0.0040	0.0418	0.0549	35	20	105	0.060	50	0.22
4	0.0288	0.00451	-1	-3	1	4,000	0.0435	0.0032	0.0382	0.0487	33	21	135	0.060	81	0.24
5	0.0321	0.00337	2	1	1	5,000	0.0412	0.0026	0.0369	0.0455	34	22	155	0.050	99	0.24
6	0.0288	0.00443	-1	-2	1	6,000	0.0391	0.0023	0.0353	0.0429	32	23	175	0.050	120	0.25
7	0.0278	0.00458	-2	-3	1	7,000	0.0375	0.0021	0.0341	0.0410	29	24	195	0.050	142	0.28
8	0.0383	0.00779	8	5	3	8,000	0.0376	0.0021	0.0342	0.0410	34	27	215	0.050	154	0.27
9	0.0342	0.00614	4	2	2	9,000	0.0372	0.0020	0.0340	0.0405	36	29	230	0.045	165	0.27
10	0.022	0.0067	-8	-10	2	10,000	0.0357	0.0019	0.0326	0.0388	26	31	245	0.045	188	0.33
11	0.0265	0.00362	-4	-4	1	11,000	0.0349	0.0017	0.0320	0.0378	22	32	255	0.040	201	0.36
12	0.0177	0.00457	-12	-13	1	12,000	0.0334	0.0016	0.0307	0.0362	9	33	265	0.040	224	0.48
13	0.0343	0.00722	4	2	2	13,000	0.0335	0.0016	0.0308	0.0362	11	35	275	0.040	229	0.46
14	0.0315	0.00678	2	0	2	14,000	0.0334	0.0016	0.0308	0.0360	11	36	280	0.035	233	0.47
15	0.0281	0.00647	-2	-3	2	15,000	0.0330	0.0015	0.0305	0.0355	7	38	285	0.035	240	0.51
16	0.0298	0.00584	0	-1	1	16,000	0.0328	0.0015	0.0304	0.0353	6	39	285	0.030	240	0.53
17	0.0377	0.00621	8	6	1	17,000	0.0331	0.0014	0.0307	0.0355	12	40	285	0.030	232	0.47
18	0.0265	0.00571	-4	-5	1	18,000	0.0327	0.0014	0.0304	0.0350	8	41	285	0.030	236	0.51
19	0.0237	0.00574	-6	-7	1	19,000	0.0323	0.0014	0.0300	0.0345	0	43	280	0.025	237	0.60
20	0.0228	0.00585	-7	0	-7	20,000	0.0318	0.0013	0.0296	0.0340	0	36	275	0.025	239	0.74

Parameters:

Varying payment error rate	
Standard deviation	0.006
Beta	40
rho	0.7
Sample size, n'	360
Annual Federal contribution	1,000

Note: *** indicates that the coefficient of variation is 10 or greater.

Table H-16. Federal withholding, Rule D, Example 16

Year	Error rate	sigma	AFDC	Current Disallowance Cash	Current Disallowance Book	Cumulated values: Federal contrib.	Error rate	sigma	Lower bound	Upper bound	Disallowance Cash	Disallowance Book	Desired Disall.	True error rate	Disall. Error	cv
0																
1	0.0721	0.02026	42	9	33	1,000	0.0721	0.0203	0.0387	0.1054	9	33	40	0.070	-2	0.48
2	0.0364	0.0124	6	1	6	2,000	0.0542	0.0119	0.0347	0.0738	9	39	75	0.065	27	0.49
3	0.054	0.01207	24	19	5	3,000	0.0541	0.0089	0.0395	0.0687	29	44	105	0.060	33	0.37
4	0.0283	0.00685	-2	-3	1	4,000	0.0477	0.0069	0.0364	0.0590	25	45	135	0.060	64	0.39
5	0.0378	0.01181	8	4	4	5,000	0.0457	0.0060	0.0359	0.0556	29	49	155	0.050	76	0.38
6	0.0144	0.01481	-16	-21	6	6,000	0.0405	0.0056	0.0313	0.0496	8	55	175	0.050	112	0.53
7	0.0486	0.01483	19	13	5	7,000	0.0416	0.0052	0.0331	0.0502	21	60	195	0.050	113	0.45
8	0.0302	0.01392	0	-4	4	8,000	0.0402	0.0049	0.0322	0.0483	17	64	215	0.050	133	0.48
9	0.0393	0.01669	9	4	6	9,000	0.0401	0.0047	0.0323	0.0479	21	70	230	0.045	139	0.47
10	0.0265	0.00811	-4	-5	1	10,000	0.0388	0.0043	0.0316	0.0459	16	71	245	0.045	157	0.49
11	0.0081	0.00709	-22	-16	-6	11,000	0.0360	0.0040	0.0294	0.0425	0	66	255	0.040	189	0.67
12	0.0172	0.01004	-13	0	-13	12,000	0.0344	0.0037	0.0282	0.0406	0	53	265	0.040	212	0.85
13	0.0198	0.00673	-10	0	-10	13,000	0.0333	0.0035	0.0275	0.0390	0	43	275	0.040	232	1.07
14	0.0268	0.0146	-3	0	-3	14,000	0.0328	0.0034	0.0272	0.0384	0	39	280	0.035	241	1.21
15	0.0306	0.0107	1	0	1	15,000	0.0327	0.0033	0.0273	0.0380	0	40	285	0.035	245	1.22
16	0.0197	0.01019	-10	0	-10	16,000	0.0319	0.0031	0.0267	0.0370	0	30	285	0.030	255	1.68
17	0.0357	0.01192	6	0	6	17,000	0.0321	0.0030	0.0271	0.0371	0	35	285	0.030	250	1.45
18	0.0271	0.00865	-3	0	-3	18,000	0.0318	0.0029	0.0270	0.0366	0	33	285	0.030	252	1.60
19	0.0094	0.00441	-21	0	-21	19,000	0.0306	0.0028	0.0261	0.0352	0	12	280	0.025	268	4.36
20	0.0196	0.00892	-10	0	-10	20,000	0.0301	0.0027	0.0257	0.0344	0	2	275	0.025	273	***

Parameters:

Varying payment error rate	
Standard deviation	0.012
Beta	40
rho	0.7
Sample size, n'	120
Annual Federal contribution	1,000

Note: *** indicates that the coefficient of variation is 10 or greater.

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APPENDIX I

EFFECT OF SUBSTITUTING $\bar{t}$ FOR $\bar{T}$ IN ESTIMATING OVERPAYMENT ERROR RATES

The estimator of the overpayment error rate in current use, $\hat{R}$, given by Equation (1) in Chapter 1 of the report and by Appendix B, involves the quantity $\bar{t}$, the average AFDC payment per case as estimated from the state sample. In the original proposal for the regression estimator, $\bar{T}$, the average AFDC payment per case in the complete caseload of the state in the specified time period was used instead of $\bar{t}$, the estimate of $\bar{T}$ from the state sample. This raises questions with regard to the statistical efficiency of the estimator $\hat{R}$, based on $\bar{t}$, and the validity of the estimator of its variance. This appendix examines these questions.

The evaluation was done by simulating the sampling and estimating procedures for Population A. For each of three sample sizes, 1000 samples were drawn.[1] In each of the samples, the regression estimator and three difference estimators (using three values of the coefficient k; see Appendix B) were computed, using $\bar{t}$ and also using $\bar{T}$. The variation of the estimates over the 1000 samples provided estimates of the variances of the alternative estimators, denoted $\sigma^2_{\bar{x}''/\bar{t}}$ and $\sigma^2_{\bar{x}''/\bar{T}}$. The results are shown in Table I-1. For both the regression and the difference estimators, the variances of the estimates of R do not differ greatly. For the regression estimator the relative difference is only 8 to 10 percent, which corresponds to a relative difference in the standard errors of only about 4 or 5 percent.

[1]The sample sizes used for these simulations were different, and generally smaller, than those used in later simulations. The reason was that these and certain other simulations were done early, with sample sizes more representative of six month samples, chosen to illustrate what happens with relatively smaller samples than the annual samples currently in use.

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Moreover, the variances of the estimates that use $\bar{t}$ are moderately smaller than of those that use $\bar{T}$. This is because the coefficient of variation of $\bar{t}$ is small and the estimated average overpayment per case, $\bar{x}''$, is positively correlated with the average AFDC payment per case. The relative variance of the ratio of two random variables u and v is given by

$$V^2_{u/v} \doteq V^2_u + V^2_v - 2\,\rho\, V_u V_v \,.$$

Here, ρ denotes the correlation between u and v. If the denominator v is a constant (which is the case when $\bar{T}$ is used), then the relvariance of the ratio reduces to V^2_u since V_v=0. If the denominator v is not a constant but a variable (which is the case when $\bar{t}$ is used), the relvariance of the ratio depends upon the value of the quantity $V^2_v - 2\rho V_u V_v$. The use of a variable v will produce a smaller variance than the use of a constant v if $\rho > V_v/V_u$. Since in our case the coefficient of variation of $\bar{t}$ is far less than the coefficient of variation of $\bar{x}''$, it does not require a very large value of the correlation ρ to give the use of $\bar{t}$ a small advantage. Consequently, we have the fortunate result that the more convenient estimator has a somewhat smaller variance and is not only appropriate but recommended.

Table I-1. Comparison of variances of $\bar{x}''/\bar{t}$ and $\bar{x}''/\bar{T}$ for Population A (Variances are shown times 10^4)

Sample size (n/n')	Estimator	$\sigma^2_{\bar{x}''/\bar{T}}$ (1)	$\sigma^2_{\bar{x}''/\bar{t}}$ (2)	Ratio (1)/(2)
1200/180	Regression	1.397	1.297	1.08
	Difference, k=1	1.383	1.307	1.06
	Difference, k=.9	1.393	1.309	1.06
	Difference, k=.8	1.445	1.351	1.07
500/80	Regression	3.136	2.897	1.08
	Difference, k=1	3.004	2.938	1.02
	Difference, k=.9	3.004	2.940	1.02
	Difference, k=.8	3.117	3.030	1.03
300/50	Regression	5.176	4.696	1.10
	Difference, k=1	4.923	4.786	1.03
	Difference, k=.9	4.981	4.791	1.04
	Difference, k=.8	5.209	4.937	1.06